M2 Guided version

01 | A strange calculation

Introduction to problem

1. Choose a number.
2. Multiply it by 3.
3. Add 6.
4. Divide this result by 3.
5. Subtract the number chosen in step 1 from the answer in step 4.

Exploration

Analyse some examples by completing the table below. You can use a spreadsheet or a calculator to do the calculations.

	Example 1	Example 2	Example 3	Example 4
Choose a number.				
Multiply it by 3.				
Add 6.				
Divide by 3.				
Subtract the number in the first line from the number in the 4th line.				

Conjecture

Describe the pattern observed.

If we carry out the algorithm described above, we always obtain

____.

A proof?

Is this conjecture always true? We can **prove** it using the techniques of **algebra**.

Let's call n the number chosen at the beginning.

Let's repeat the calculations made during the exploration (but using n):

Choose a number.	n
Multiply it by 3.	
Add 6.	
Divide by 3.	
Subtract the number in the first line from the number in the 4th line.	
Simplify the expression.	

The final result is ____, regardless of the number n we start with.

Conclusion

	✓ ? ✗
If we run the following algorithm: 1. Choose a number. 2. Multiply it by 3. 3. Add 6. 4. Divide this result by 3. 5. Subtract the number at the beginning of step 1 from the answer in step 4. the end result is always __________.	

02 | Happy Birthday!

Introduction to problem

Take the number of the month of your birthday (1 for January, 2 for February, ...) and multiply it by 2 . Add 5 , then multiply the result by 50 . Add the day of the month of your birthday. Subtract 250 to get a 4- or 3-digit number.

Exploration

Analyse some examples by completing the table below. You can use a spreadsheet or a calculator to do the calculations.

	Student 1	Student 2	Student 3	Student 4
Take the month of your birthday.				
Multiply by 2.				
Add 5.				
Multiply by 50.				
Add the day of the birthday.				
Subtract 250.				

Compare the final results with the birthdays that you started with.

Conjecture

Complete the following text to describe the phenomenon observed:

The final result shows _______________ of a birthday as follows:

6. The number formed by the _______________ digit and the _______________ digit of the final result represents the month of the birthday.
7. The number formed by the ______________ digit and the _________ digits of the final result represents the _______________ of the birthday.

A proof?

Does this rule always work? There are two ways to find out:

Option 1:

Try **all** possible **dates**: from 01.01 to 31.12. How many cases do you need to check?

To assess such a large number of different cases, it is advisable to use a **spreadsheet** or a **programming language** to automate the calculations.

After assessing all the cases, we found that

Option 2:

We'll try to **prove** the conjecture using the techniques of **algebra**.

Let's call the day of a person's birthday d.

Let's call a person's month m.

Take the month of the birthday.	m
Multiply by 2.	
Add 5.	
Multiply by 50.	
Add the day d of the birthday.	
Subtract 250.	
Simplify the expression obtained.	

The final result is _______________ .

The last two digits of this result represent _____________________________________.

The first digit or first two digits of this result represent _______________________.

Conclusion

If we follow the following algorithm :

Take the number of the month of your birthday.
Multiply by 2.
Add 5.
Multiply by 50.
Add the day of the month of your birthday.
Substract 250.

this gives :

- a three-digit number if the month of the birthday is less than or equal to _____. The hundreds digit of this number represents _________ and the last two digits represent the _________ of the birthday.
- a four-digit number if the month of the birthday is greater than or equal to _____. The first two digits of this number represent _________ and the last two digits represent the _________ of the birthday.

03 | Multiply by 9

Introduction to problem

Choose a number between 1 and 10. Multiply it by 9. Add the digits of the new number and add 4. What happens?

Exploration

Analyse a few examples.

	Example 1	Example 2	Example 3	Example 4	Example 5
Choose a number between 1 and 10.	1	2	3	4	5
Multiply by 9.					
Add up the digits.					
Add 4.					

We can see that the result of this algorithm always seems to be equal to ___________.

Conjecture

Based on the results of the previous exploration, try to formulate a general rule.

If we take a number between 1 and 10, multiply it by 9, then calculate the sum of the digits in the result and finally add 4, we obtain the number _______.

A proof?

In fact, we only have _____ other cases to check. If we manage to check our rule for these cases, our conjecture is proven:

Choose a number between 1 and 10.					
Multiply it by 9.					
Add up the digits.					
Add 4.					

Does the rule also work for numbers greater than 10? Try some examples:

Choose a number greater than 10					
Multiply it by 9.					
Add up the digits.					
Add 4.					

What can you conclude?

The rule	☐ ☐	works does not work	for numbers greater than 10.

Conclusion

If we take any integer between 1 and 10 and follow the following algorithm: Multiply it by 9. Add up the digits. Add 4. the end result is always _________.	

04 | Multiply by 6

Introduction to problem

Take an even number and multiply it by 6. Compare the units digit of the result with the units digit of the number you started with. What do you find?

Exploration

Analyse a few examples.

	Example 1	Example 2	Example 3	Example 4
Choose an even number.	4	12		
Multiply it by 6.				
Units digit of the result.				
Units digit of the number chosen at the start.				

It seems that the units digit of the result of this algorithm is always equal to ____.

Conjecture

Based on the results of the previous exploration, try to formulate a general rule:

If an even number is multiplied by 6, then the units digit of the result equals ____.

A proof?

Let's call the number at the beginning n.

Since n is even, n can be written as...	
Multiply it by 6.	
Simplify the expression.	

The final result is _____________.

Write this result as the sum of two terms:

_________ = __________ + _________

The units digit is shown here.

The units digit equals ____.

Conclusion

We have proved the following theorem:

If an even number is multiplied by 6, then the units digit of the result equals ____.	

05| Three digits become six digits

Introduction to problem

1. Choose a three-digit number and write it twice to make a six-digit number. For example, 371371 or 552552.
2. Divide the number by 7.
3. Divide the result by 11.
4. Divide the result by 13.

Exploration

Analyse some examples by completing the table below. You can use a spreadsheet or a calculator to do the calculations.

	Example 1	Example 2	Example 3	Example 4
Choose a three-digit number				
Write it down twice to get a 6-digit number.				
Divide by 7.				
Divide by 11.				
Divide by 13.				

Conjecture

Describe the phenomenon observed :

If we choose a 3-digit number and run the algorithm described above, we get ____.

?

A proof?

Does this rule still work? To check, we can use the techniques of **algebra**.

Let's call the 3-digit number at the beginning n.

Step 1:

Let's construct the 6-digit number obtained by repeating the number n twice.

Hint: Use your calculator. Type in the 3-digit number n. What calculation can you do so that the calculator displays the 6-digit number made up of twice the 3-digit number?

The 6-digit number is written as follows (using n): _________________________

Step 2:

Let's continue running the algorithm.

Choose a three-digit number.	n
Write it down twice to get a 6-digit number.	
Divide by 7.	
Divide by 11.	
Divide by 13.	
Write the expression as a fraction.	
Simplify the fraction as far as possible.	

The resulting number is : ____

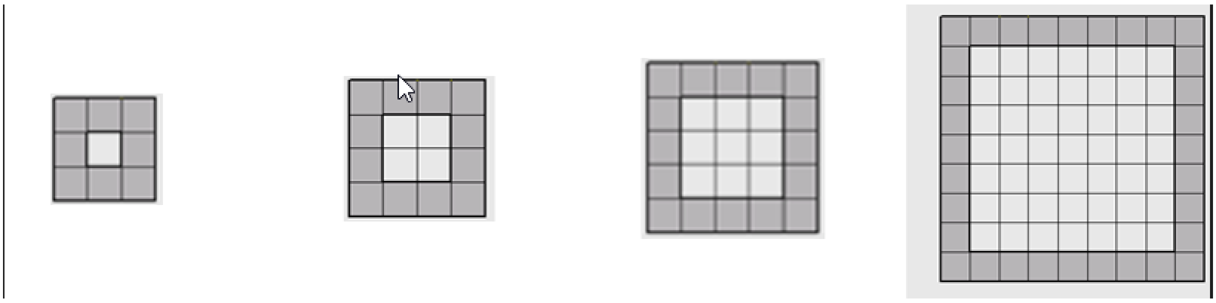
Conclusion

	✓ ? ✗
We choose a three-digit number and apply the following algorithm: 1. Write it down twice to get a six-digit number. 2. Divide the number by 7. 3. Divide the result by 11. 4. Divide the result by 13. Then we always get ____ .	

06 | Mosaic tiles

Introduction to problem

Here are some mosaic tiles consisting of white tiles forming squares of different sizes. Every white square is surrounded by a frame of grey tiles. Here are four examples.



Count the grey squares. What do you notice?

Exploration

Analyse some examples by completing the table below.

Number of white tiles along one edge	Drawing	Number of grey tiles
4		
5		
6		

Conjecture

Describe the phenomenon observed :

The number of grey squares surrounding a square of n by n white squares is equal to ________.

A proof?

Does this rule still work? To check, we can use the techniques of **algebra**.

Let's call n the size of the white square.

Number of white tiles along one edge of the square.	n
Number of tiles along one edge in the white-and-grey square.	
The number of grey tiles.	
Simplify.	

If the white square is of size $n \times n$, the number of grey squares is __________________________.

Conclusion

	✔ ? ✖
The number of grey squares surrounding a square with n white tiles along one edge is equal to ________.	

07 | The calendar

Introduction to problem

The numbers for each month in the calendar can be grouped into squares of different sizes:

January

S	M	T	W	T	F	S
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30
31						

February

S	M	T	W	T	F	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29					

March

S	M	T	W	T	F	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

April

S	M	T	W	T	F	S
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

May

S	M	T	W	T	F	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

June

S	M	T	W	T	F	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30		

Step 1: Draw a few squares on the calendar.

- Start by drawing squares of 2 by 2 days.
- Then try squares of 3 by 3 days.

Step 2: For each square you have drawn:

- Multiply the number in the top right-hand corner by the number in the bottom left-hand corner.
- Multiply the number on the top left with the number on the bottom right.
- Calculate the difference between these two products.

Here are two examples:

February

S	M	T	W	T	F	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29					

$$11 \cdot 17 - 10 \cdot 18 = 7$$

April

S	M	T	W	T	F	S
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

$$14 \cdot 26 - 12 \cdot 28 = 28$$

What do you notice?

Exploration

Draw squares of different sizes in the following calendar. Then complete the table.

janvier						1
L	M	M	J	V	S	D
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

février						2
L	M	M	J	V	S	D
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28

mars						3
L	M	M	J	V	S	D
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

avril						4
L	M	M	J	V	S	D
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30		

mai						5
L	M	M	J	V	S	D
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30
31						

juin						6
L	M	M	J	V	S	D
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30				

juillet						7
L	M	M	J	V	S	D
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

août						8
L	M	M	J	V	S	D
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30	31					

septembre						9
L	M	M	J	V	S	D
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30			

octobre						10
L	M	M	J	V	S	D
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

novembre						11
L	M	M	J	V	S	D
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30					

décembre						12
L	M	M	J	V	S	D
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

Square size	Number top right corner	Number bottom left corner	Product of the two	Number top left corner	Number bottom right corner	Product of the two	Difference
2							
2							
2							

3							
3							
3							
4							
4							
4							

Conjecture

Describe the phenomenon observed :

We see that the difference between the product of the number in the top right-hand corner with the number in the bottom left-hand corner and the product of the number in the top left-hand corner with the number in the bottom right-hand corner of a square of n by n days is equal to

_________________.

A proof?

Does this rule still work? To check, we can use the techniques of **algebra**.

Let's call n the size of the square and let's call a the number in the top left-hand corner.

Top left corner	a
Top right corner	
Bottom left corner	
Bottom right corner	
Product of the number in the top right corner with the number in the bottom left corner	
Product of the number in the top left corner and the number in the bottom right corner	
Difference between the two products	
Simplify the expression.	

Conclusion

	✔ ? ✘
The difference between the product of the number in the top right corner with the number in the bottom left corner and the product of the number in the top left corner with the number in the bottom right corner of a square of size n is equal to _______________.	

08 | Sum of odd numbers

Introduction to problem

$$1 = 1$$
$$1 + 3 = 4$$
$$1 + 3 + 5 = 9$$
$$1 + 3 + 5 + 7 = 16$$
$$1 + 3 + 5 + 7 + 9 = 25$$
$$\dots$$
$$1 + 3 + 5 + \cdots + 197 + 199 = 100\,000$$

Do you spot a pattern?

Exploration

Use a spreadsheet or calculator to complete the following table:

Step k		
1	$1 =$	$=$
2	$1 + 3 =$	$=$
3	$1 + 3 + 5 =$	$=$
4	$1 + 3 + 5 + 7 =$	$=$
5	$1 + 3 + 5 + 7 + 9 =$	$=$
6	$1 + 3 + 5 + 7 + 9 + 11 =$	$=$
7	$1 + 3 + 5 + 7 + 9 + 11 + 13 =$	$=$

Conjecture

Based on the results of the exploration above, try to formulate a general rule:

The sum of k odd numbers is ____________________ and is equal to

_________.

A proof?

We are going to prove the conjecture for sums up to 100. There are_________ cases to check. As this is a relatively large number, it is reasonable to use a **spreadsheet** or a **programming language**:

In this case, we suggest you use a spreadsheet program (such as Excel, Numbers, Google Sheets, etc.) and draw up a list similar to the one below:

	A	B	C	D	E
1–2	Step k	k-th odd number	sum of the first k odd numbers	Conjecture	Check
3	1	1	1	1	TRUE
4	2	3	4	4	TRUE
5	3	5	9	9	TRUE
6	4	7	16	16	TRUE
7	5	9	25	25	TRUE
8	6	11	36	36	TRUE
9	7	13	49	49	TRUE
10	8	15	64	64	TRUE
11	9	17	81	81	TRUE
12	10	19	100	100	TRUE
13	11	21	121	121	TRUE
14	12	23	144	144	TRUE
15	13	25	169	169	TRUE

To check all the possible cases, you need to work with **formulas**:

In column A, start by writing 1 in box A3. Then in box A4, write a formula that adds 1 to the box above (A3+1). Then drag this formula down to fill the whole column. The result is: 1, 2, 3, 4...

	A	B	C	D	E
1–2	Step k	k-th odd number	sum of the first k odd numbers	Conjecture	Check
3	1				
4	=A3+1				
5					

In column B, we also write 1 in box B3. But in box B4, we add 2 to the box above (B3+2). Pull this formula downwards. You get: 1, 3, 5, 7...

	A	B	C	D	E
1 2	Step k	k-th odd number	sum of the first k odd numbers	Conjecture	Check
3	1	1			
4	2	=B3+2			
5	3				

For column C, in box C4, add the value of C3 to that of B4. Then pull this formula down. This column allows us to see how the numbers add up.

	A	B	C	D	E
1 2	Step k	k-th odd number	sum of the first k odd numbers	Conjecture	Check
3	1	1	1		
4	2	3	=C3+B4		
5	3	5			

In column D, we're going to test our idea: we write our formula in box D3 and pull it down.

Column E will be used to check whether our idea is correct or not.

	A	B	C	D	E
1 2	Step k	k-th odd number	sum of the first k odd numbers	Conjecture	Check
3	1	1	1	1	=C3=D3
4	2	3	4	4	

This makes it easy to check all the cases for k from1 to _________.

Conclusion

	✓ ? ✗
The sum of k odd numbers is ________ and is equal to ________ (for k from1 to _________).	

Does the conjecture hold true for values of k greater than 100 ?

Search the Internet or ask artificial intelligence for help in finding answers!

09 | Add 41

Introduction to problem

Choose a natural number n . Multiply it by itself. Add to the result the number you chose at the start, then add 41. Start with $n = 0$, then $n = 1$ and so on. What do you find?

Exploration

Analyse some examples.

n	Multiply n by itself	Add n	Add 41	Final result	Divisors of the final result
0					
1					
2					
3					
4					
5					

The final results are all ___________________________numbers.

Conjecture

Based on the results of the previous exploration, try to formulate a general rule.

If you multiply a number by itself, then add the number you chose at the start and finally add 41, you get ___________________________.

A proof?

Try to explain why your rule always works or look for counterexamples. You can use the technological tools at your disposal.

Check your conjecture up to n=40 and n=41. Use a spreadsheet (or do it by hand if you like). What do you find?

Conclusion

Write a short commentary summarising your observations and thoughts.

10 | 3s followed by a 1

Introduction to problem

What are the divisors of 31? 331? And 3331? What do you find?

Exploration

Analyse a few examples.

Number	Dividers
31	{1, 31}
331	
3331	

Conjecture

Based on the results of the previous exploration, try to formulate a general rule:

All numbers of the form 333...1 are ________________.

A proof?

Try to explain why your rule always works or look for counterexamples. You can use the technological tools at your disposal.

Check your guess up to step 8. Use a spreadsheet (or do it by hand if you like). What do you find?

Conclusion

Write a short commentary summarising your observations and thoughts.

11 | Some rather unusual powers of 2

Introduction to problem

Take a natural number n . Raise 2 to the power of n to obtain the result m. Then raise 2 to the power m. Add 1 to the result.

Start with $n = 0$, then move on to $n = 1$ and so on. What do you notice?

Exploration

Analyse some examples:

n	2^n	2^{2^n}	$2^{2^n} + 1$
0	$2^0 = 1$	$2^1 = 2$	$2 + 1 = 3$
1	$2^1 = 2$	$2^2 = 4$	$4 + 1 = 5$

Conjecture

Based on the results of the previous exploration, try to formulate a general rule:

All numbers of the form $2^{2^n} + 1$ are ______________________________.

?

A proof?

Try to explain why your rule always works or look for counterexamples. You can use the technological tools at your disposal.

Check your conjecture up to $n = 5$. Use a calculator or spreadsheet to do the calculations. Use artificial intelligence or a search engine to check your guess. What do you find?

Conclusion

Write a short commentary summarising your observations and thoughts.

12 | Finding common divisors

Introduction to problem

Take a natural number n. Calculate the value of $A = n^2 + 7$ and $B = (n+1)^2 + 7$, then try to find common divisors of A and B. Start with $n = 0$, then move on to $n = 1$ and so on. What do you find?

Exploration

Analyse some examples.

n	$A = n^2 + 7$	$B = (n+1)^2 + 7$	gcd(A,B)
0	7	8	1
1	8	11	1

Conjecture

Based on the results of the previous exploration, try to formulate a general rule.

All numbers of the form $n^2 + 7$ and $(n+1)^2 + 7$ are

________________________________.

Any proof?

Try to explain why your rule always works or look for counterexamples. You can use the technological tools at your disposal.

Check your conjecture up to $n = 14$. Use a calculator or spreadsheet to do the calculations. What do you find?

Conclusion

Write a short commentary summarising your observations and thoughts.

13 | Even numbers and prime numbers

Introduction to problem

$$\begin{aligned} 4 &= 2+2 \\ 6 &= 3+3 \\ 8 &= 5+3 \\ 10 &= 7+3 \\ 12 &= 7+5 \\ 14 &= 7+7 \\ 16 &= 11+5 \\ &\dots \\ 100 &= 41+59 \end{aligned}$$

Can you spot a pattern?

Exploration

Here are a few more examples:

$16 = 11 + 5$	11 and 5 are ________________ numbers.
$18 =$	________ and ________ are ________________ numbers.
$20 =$	________ and ________ are ________________ numbers.
$22 =$	________ and ________ are ________________ numbers.
$24 =$	________ and ________ are ________________ numbers.
$26 =$	________ and ________ are ________________ numbers.
$28 =$	________ and ________ are ________________ numbers.
$30 =$	________ and ________ are ________________ numbers.

Conjecture

Based on the results of the previous exploration, try to formulate a general rule.

Any _______________ number less than or equal to100 can be written as

the sum of two _______________ numbers.

Any proof?

Try to explain why your rule always works or look for counterexamples. You can use the technological tools at your disposal.

Search the Internet or ask artificial intelligence for help in finding answers!

You can also use the following script to find solutions:

main.py > ...

```python
import sympy

a=int(input('Enter an even number:'))

for p in sympy.primerange(0, 200):
  if sympy.isprime(a-p):
    print(a,'=',p,'+',a-p)

```

Can you rewrite any even number less than or equal to 100 as the sum of two primes?
☐ Yes ☐ No

A counterexample?

____ .

Are there any even numbers greater than 100 that cannot be rewritten as the sum of two primes?

____ .

Conclusion

	✅ ❓ ❌
Any _______________ number less than or equal to100 can be written as the sum of two_______________numbers.	
Any _______________ number can be written as the sum of two _______________ numbers. This problem is known as the __________________________.	

14 | An algorithm that always finishes?

Introduction to problem

Choose a strictly positive integer.

- If it is even, divide by 2.
- If it is odd, multiply it by 3 and add 1 to the result.

Repeat this operation a large number of times to make the result as small as possible.

Exploration

Analyse some examples.

Natural number between 1 and 100	Intermediate results	Smallest result
1	$4; 2; 1; 4; 2; 1; \ldots$	1
2	$1; 4; 2; 1; 4; 2; 1; \ldots$	1
3	$10; 5; 16; 8; 4; 2; 1; 4; 2; 1; \ldots$	1
4		
5		
6		
7		
8		
9		
10		

Conjecture

Based on the results of the previous exploration, try to formulate a general rule.

For integers chosen between 1 and 100, the algorithm described above always results in the value _______ after a finite number of iterations.

A proof?

Try to explain why your rule always works or look for counterexamples. You can use the technological tools at your disposal.

Search the Internet or ask artificial intelligence for help in finding answers!

You can also test the conjecture on several cases on the following website: https://www.dcode.fr/conjecture-syracuse

For all numbers between 1 and 100 , does the algorithm always arrive at the number 1 after a finite number of iterations? ☐ Yes ☐ No Is there a counterexample? ____ . What if you take starting numbers greater than 100? ____ ____

Conclusion

	✅ ❓ ❌
For integers chosen between 1 and 100, the algorithm described above always results in the value ________ after a finite number of iterations.	
For **any positive integer**, the algorithm described above always results in the value ________ after a finite number of iterations. This problem is known as the ______________________________.	

15 | A sum of 4 cubes

Introduction to problem

$$
\begin{aligned}
1 &= 1^3 + 0^3 + 0^3 + 0^3 \\
2 &= 1^3 + 1^3 + 0^3 + 0^3 \\
3 &= 1^3 + 1^3 + 1^3 + 0^3 \\
4 &= 1^3 + 1^3 + 1^3 + 1^3 \\
5 &= 2^3 + (-1)^3 + (-1)^3 + (-1)^3 \\
&\dots \\
13 &= 10^3 + 7^3 + 1^3 + (-11)^3
\end{aligned}
$$

What do you notice?

Exploration

Analyse some examples.

$6 =$	$8 - 1 - 1 + 0$	$=$
$7 =$	$8 - 1 + 0 + 0$	$=$
$8 =$		$=$
$9 =$		

Conjecture

Based on the results of the previous exploration, try to formulate a general rule.

Any integer between 1 and 13 can be written as

____.

Any proof?

Try to explain why your rule always works or look for counterexamples. You can use the technological tools at your disposal.

	Do some research on the Internet or ask artificial intelligence for help to check the remaining cases!
$10 =$	$=$
$11 =$	$=$
$12 =$	$=$
For all numbers between 1 and 13, can they be rewritten as ___________________________________ ? ☐ Yes ☐ No Is there a counterexample? ____ . What if you use numbers greater than 13? ____ ____	

Conclusion

	✔ ? ✖
Any integer between 1 and 13 can be written as ____.	
Any **positive integer** can be written as ____. This problem is known as the ________________________________.	