2.3 Teaching materials

M1 Barcodes are everywhere! But where exactly?

Look around you. Can you see any barcodes? If so, photograph them and upload them to the following platform:

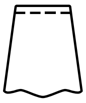
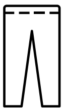
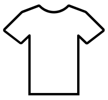
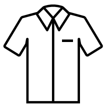
https://flinga.fi/s/FV6B92C

Do you know of any other places where barcodes can be found? What role do they play? Why are they used?

M2 The database

Barcodes are often linked to a **database** that enables the various codes to be associated with other information about the scanned object.

In a clothing shop, all products are marked with a unique barcode that is linked to the product price. The reader reads the code and identifies the product. The system's database is used to assign a price to each product.

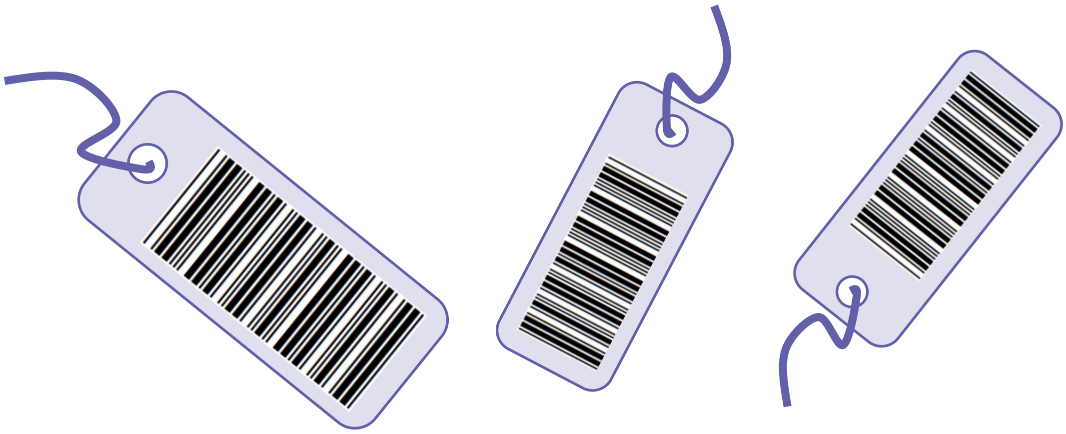
Code	Product	Prices
10011		50 €
11100		30 €
01100		45 €
00011		60 €
00100		15 €
01101		100 €
11001		30 €
01010		10 €
11101		100 €
00010		80 €
10101		50 €

At the checkout

- Beyoncé buys the following items. How much will she have to pay?

Beyoncé has to pay ________________________________ .

- Justin takes the following items to the checkout. How much does he have to pay?



Justin has to pay ________________________________ .

A few thoughts

- Does the layout or orientation of barcodes affect the scan quality?

____

- Products are coded using a code of _____ digits made up of the digits _____ and _____.

M3 Testing the limits of the barcode reader

During the transport of goods or following accidents, the label bearing the barcode may be damaged or altered.

Try scanning the various damaged barcodes. Can your reader recognise all the codes?

Indicate by ✓ if the code is readable and by ✗ if it is not.

Damaged label	Description of the damage	Legible?
1 234567 891231		
9 876543 219879		
3 551780 840111		

Damaged label	Description of the damage	Legible?
4 00 817 512029		
7 622201 7918 4		
5 41 038 1 1279		
9 82359 3 37 3		

What can you conclude?

M4 The check digit

In the previous activity, you noticed that the reader can even identify damaged codes, provided the damage is not too extensive. In fact, the reader can read the damaged code when there is **at most one digit** that **is** not legible on the label. This is made possible by a **check digit** contained in the code, which enables a missing digit to be recovered if necessary. The idea of the check digit is to add an extra digit to the code (normally at the end of the code) which is **calculated from the other digits in the code** using a certain formula or mathematical algorithm.

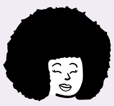
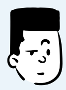
The check digit can also be used to **check** whether the code has been **entered correctly** into the computer system or whether it is a **faulty code**.

How do you calculate the check digit?

Let's take a code made up of 4 digits chosen from the elements in the set

$$\boldsymbol{A} = \{\mathbf{0}; \mathbf{1}; \mathbf{2}; \mathbf{3}; \mathbf{4}; \mathbf{5}; \mathbf{6}; \mathbf{7}; \mathbf{8}; \mathbf{9}\}$$

Alice and **Bob** propose two different methods for calculating the check digit:

Alice	**Bob**
 I calculate the **sum of** the first 3 digits of the code. The check digit will be the **last digit** of the result obtained.	 I calculate the **product of** the first 3 digits of the code. The check digit will be the **last digit** of the result obtained.

Exercise: Take a look at some examples:

Code	Alice			Bob		
	Sum of digits	Check digit	Full code	Product of digits	Check digit	Full code
115	7	7	1157	5	5	1155
602						
542						
922						
859						

Do both methods work to generate barcodes?

Exercise: Are the codes correct?

Mark correct codes with ✅ and incorrect codes with ❌ :

Alice's codes

Full code	Code without check digit	Check digit calculated using the method	Correct or incorrect?
1236			
0461			
4734			
9876			

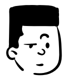
Bob's codes

Full code	Code without check digit	Check digit calculated using the method	Correct or incorrect?
1236			
0461			
4734			
9876			

Can the check digit be used to check that the code has been entered correctly? In both methods?

M5 A damaged code

Alice and Bob have a problem: unfortunately, one of their codes has become unreadable! Can you help them find the original code?

Alice's damaged code	**Bob**'s damaged code
 $23x2$	 $23x2$

Remember:

- Alice calculates the **sum of the first 3 digits** of the code and the check digit (i.e. the 4th digit) is the **last digit** of the result.
- Bob calculates the **product of the first 3 digits** of the code and the check digit (i.e. the 4th digit) is the **last digit** of the result.

Alice's damaged code

The sum of the digits is :

$S =$ ____________________________

The **last digit** of S is 2.

x takes its values from the set

$A = \{0; 1; 2; 3; 4; 5; 6; 7; 8; 9\}$

x	S
1	6
2	
3	
4	
5	
6	
7	
8	
9	
0	

The **last digit** of S is 2 for

$S =$ _______________

So :

$x =$ _______________

There is _________ possibility for the value of x.

Initial code: ______________________

Bob's damaged code

The **product of the digits** is :

$P =$ ___________________________

The **last digit** of P is 2.

x takes its values from the set

$A = \{0; 1; 2; 3; 4; 5; 6; 7; 8; 9\}$

x	P
1	6
2	
3	
4	
5	
6	
7	
8	
9	
0	

The **last digit** of P is 2 for

$P =$ _______________

So :

$x =$ _______________

There are _________ possibilities for the value of x.

Initial code: _______________________

M6 Calculating a missing digit

Now let's imagine that, in each code, there is **a single** digit that is unreadable by the reader. The unreadable digit is indicated by x . Will the check digit be able to find the missing digit in all cases?

Alice's damaged codes

Alice calculates the **sum of the first 3 digits** of the code and the check digit will be the **last digit** of the result.

Damaged code	Check digit	Sum of the first 3 digits	Result of the sum	Value of x	Initial code
$58x2$	2	$5+8+x=13+x$	22	9	5892
$x148$					
$1x51$					
$79x3$					

Is the value of x found in each case?

Bob's damaged codes

Bob calculates the **product of the first 3 digits** and the check digit will be the **last digit** of the result.

Damaged code	Check digit	Product of the first 3 digits	Product results	Value of x	Initial code
$x178$	8	$x \cdot 1 \cdot 7 = x \cdot 7$	28	4	4178
$1x31$					
$23x6$					
$15x3$					

Is the value of x found in each case?

What can you conclude? Tick the right answer :

- ☐ **Alice**'s system is better than **Bob**'s, because it allows a single code to be retrieved from the check digit.
- ☐ **Bob**'s system is better than **Alice**'s, because it allows several codes to be retrieved from the check digit.
- ☐ **Alice**'s and **Bob**'s systems are equivalent, as they both allow at least one initial code to be retrieved from the check digit.

M7 Reminder: Euclidean division

From a mathematical point of view, what does it mean to "take the last digit of a number" as a check digit? To explain this concept in detail, you'll need **Euclidean division**.

Definition

Euclidean division is an operation which associates two natural numbers called the **dividend** D and the **divisor** d with two other natural numbers called the **quotient** q and the **remainder** r such that :

$$D = d \cdot q + r \text{ where } 0 \leq r < d$$

Examples:

Euclidean division of 57 by 2 : 57 : 2 = 28 4 17 16 1 $57 = 2 \cdot 28 + 1$ so the remainder of the Euclidean division of 57 by 2 is equal to 1.	Euclidean division of 57 by 3 : 57 : 3 = 19 3 27 27 0 $57 = 3 \cdot 19 + 0$ so the remainder of the Euclidean division of 57 by 3 is equal to 0.
Euclidean division of 57 by 5 : 57 : 5 = 11 5 07 5 2 $57 = 5 \cdot 11 + 2$ so the remainder of the Euclidean division of 57 by 5 is equal to 2.	Euclidean division of 57 by 10 : 57 : 10 = 5 50 7 $57 = 10 \cdot 5 + 7$ so the remainder of the Euclidean division of 57 by 10 is 7.

M8 Mental arithmetic

Examples:

- $16 = 2 \cdot 8 + 0$ so the remainder of the Euclidean division of 16 by 2 is equal to 0.
- $17 = 2 \cdot 8 + 1$ so the remainder of the Euclidean division of 17 by 2 is equal to 1.

Practise doing Euclidean division **in your head**!

Euclidean division by $d = 2$

Euclidean division by $d = 2$

D	q	r
16	8	0
17	8	1
30		
75		
106		

D	q	r
0		
1		
2		
3		
4		
5		
6		
7		
8		
9		

The remainder can take the following values: ______________________________________ .

Euclidean division by $d = 3$

Euclidean division by $d = 3$

D	q	r
16	5	1
17		
30		
75		
106		

D	q	r
0		
1		
2		
3		
4		
5		
6		
7		
8		
9		

The remainder can take the following values: ____ .

Euclidean division by $d = 5$

Euclidean division by $d = 5$

D	q	r
16	3	1
17		
30		
75		
106		

D	q	r
0		
1		
2		
3		
4		
5		
6		
7		
8		
9		

The remainder can take the following values: ____ .

Euclidean division by $d = 10$

Euclidean division by $d = 10$

D	q	r
16	1	6
17		
30		
75		
106		

D	q	r
0		
1		
2		
3		
4		
5		
6		
7		
8		
9		

The remainder can take the following values: ______________________________ .

There's a trick to calculate the remainder of Euclidean division by 10 . Do you know what it is?

____

____

Use this trick to quickly calculate the remainder of the Euclidean division by 10 of the following numbers:

D	r
93	
170	
245	
568	
1021	

M9 The same remainder for different numbers

In the previous activity, we found that if we carry out the Euclidean division by a natural number d then the remainder can take the following values:

In all, the rest can take on _______ different values.

Consequently, many numbers will have **the same remainder** if we apply Euclidean division by d.

Example:

The numbers 5 and 9 have the same remainder in Euclidean division by 2.

1) $5 = 2 \cdot 2 + 1$
2) $9 = 2 \cdot 4 + 1$

Let's look at some other examples:

Euclidean division by $d = 5$
In the first row of the table below, write numbers that all have a remainder equal to 0 when divided by 5 . Do the same for remainders equal to 1, 2, 3 and 4.

Euclidean division by $d = 5$	Numbers						
$r = 0$	0	5	10				
$r = 1$	1						
$r = 2$	2						
$r = 3$	3						
$r = 4$	4						

Euclidean division by $d = 3$
Repeat the exercise for Euclidean division by 3.

Euclidean division by $d = 3$	Numbers						
$r =$	0						
$r =$							
$r =$							

M10 Congruent numbers modulo d

Definition
If two numbers a and b have **the same remainder by Euclidean division by d**, then the numbers a and b are said to be **congruent modulo d**. We write: $$a \equiv b \pmod{d}$$

Examples:

1)	$16 \equiv 21 \pmod{5}$,	because	$16 = 5 \cdot 3 + 1$	and	$21 = 5 \cdot 4 + 1$
	We write :	$16 \equiv 21 \equiv 1 \pmod{5}$			
2)	$16 \equiv 20 \pmod{2}$,	because	$16 = 2 \cdot 8 + 0$	and	$20 = 2 \cdot 10 + 0$
	We write :	$16 \equiv 20 \equiv 0 \pmod{2}$			
3)	$16 \equiv 40 \pmod{12}$,	because	$16 = 12 \cdot 1 + 4$	and	$40 = 12 \cdot 3 + 4$
	We write :	$16 \equiv 40 \equiv 4 \pmod{12}$			

Exercise 1:

According to ISO 8601, the current time is displayed in the format `hh:mm:ss` where

- `hh` represents hours with an integer value between `00` and 23 (`24:00:00` being defined as the end time)
- `mm` represents minutes with an integer value between `00` and 59
- `ss` represents seconds with an integer value between `00` and 59
- A plane leaves Paris at `18:40:00` for Santiago de Chile. The estimated duration of the direct flight is 13h30min. At what time (in the French time zone) does the plane land at Santiago de Chile airport?

- A rocket takes 30 hours to reach the International Space Station (ISS). If arrival at the ISS is scheduled for 05:00:00 (GMT+0), at what time (in GMT+0) should the rocket take off from Earth?

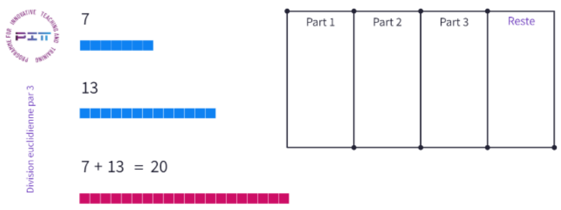
Exercise 2: Days of the week

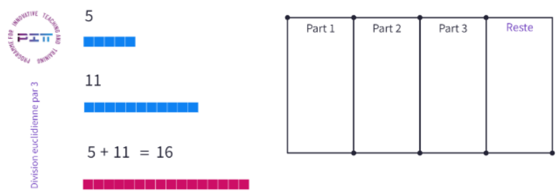
	Indicates the day of the week	Calculation
Today, it's a ...		
In 7 days, it will be a ...		
In 8 days, it will be a ...		
In 10 days, it will be a ...		
In 22 days, it will be a ...		
In 40 days, it will be a ...		
In 91 days, it will be a ...		
In 365 days, it will be a ...		

M11 Adding remainders

Imagine you have two bags of sweets that you want to distribute **equally** (i.e. everyone gets the same share) to **3 people**.

Mathigon interaction





Case 1:
The first bag contains 7 sweets and the second 13.

Alice first distributes the sweets from the 1st bag equally to the 3 people. How many sweets are left from the first bag?

a	q_1	r_1
7		

Then she distributes the sweets from the 2nd bag. How many sweets are left in the second bag?

b	q_2	r_2
13		

In total, how many sweets are left? ______________________________

Bob did things differently. He opens both bags and puts the sweets all together. Then he distributes them equally to the 3 people.

$a+b$	q	r
$7+13=20$		

How many sweets can't be distributed? ______________________________

What do you notice?

Case 2:

The first bag contains 5 sweets and the second 11.

Alice first distributes the sweets from the 1st bag equally to the 3 people. How many sweets are left from the first bag?

a	q_1	r_1
5		

Then she distributes the sweets from the 2nd bag. How many sweets are left in the second bag?

b	q_2	r_2
11		

In total, how many sweets are left? ______________________________

What can she do with the remaining sweets? How many sweets will be left? ______________________________

Bob did things differently. He opens both bags and puts the sweets all together. Then he distributes them equally to the 3 people.

$a + b$	q	r
$5 + 11 = 16$		

How many sweets can't be distributed? ______________________________

What do you notice?

____

____

____

____

Exercise :

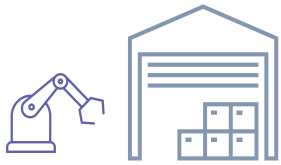
Complete the table below:

Divisor	a	r_1	b	r_2	$r_1 + r_2$	$S = a + b$	r
$d = 2$	8		5				
$d = 2$	9		7				
$d = 3$	8		14				
$d = 5$	29		18				
$d = 6$	14		10				
$d = 10$	45		37				

M12 A new encoding for Mrs Stocktout

There are **40 different items** in Mrs Stocktout's warehouse, all packed in identical boxes. The warehouse is so large that she decides to programme a robot to help her pick up and transport the items ordered. The robot is to be equipped with a barcode scanner to scan and identify the products. This means that Mrs Stocktout has to provide each item with a unique code. However, Mrs Stocktout imposes a few restrictions on the code to be used:

- She wants the code to be **exactly 2 digits** long.
- She wants to use **as few different numbers as possible**.
- The digits used are **consecutive numbers**.

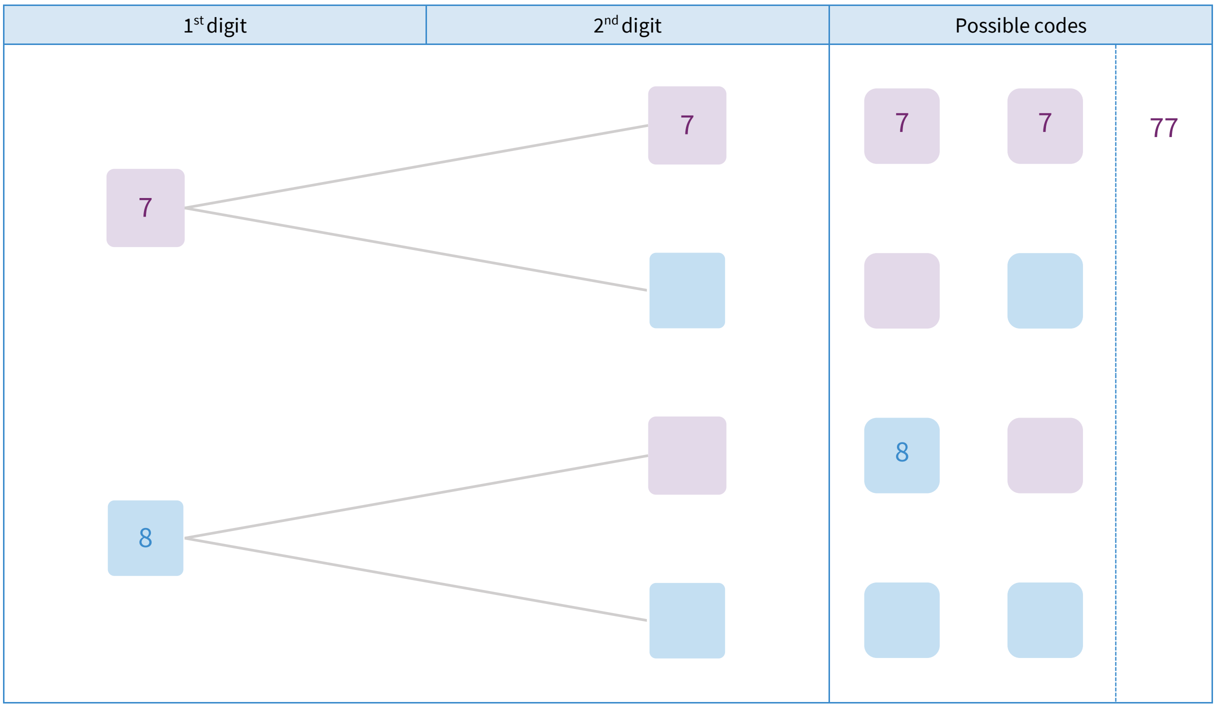


Help Mrs Stocktout create the codes for her 40 items.

Attempt 1

Mrs Stocktout chooses the first two digits from the set $A = \{7; 8\}$. What codes can she form with these digits? How many different codes are there?

A **tree diagram** can help you visualise all the combinations:



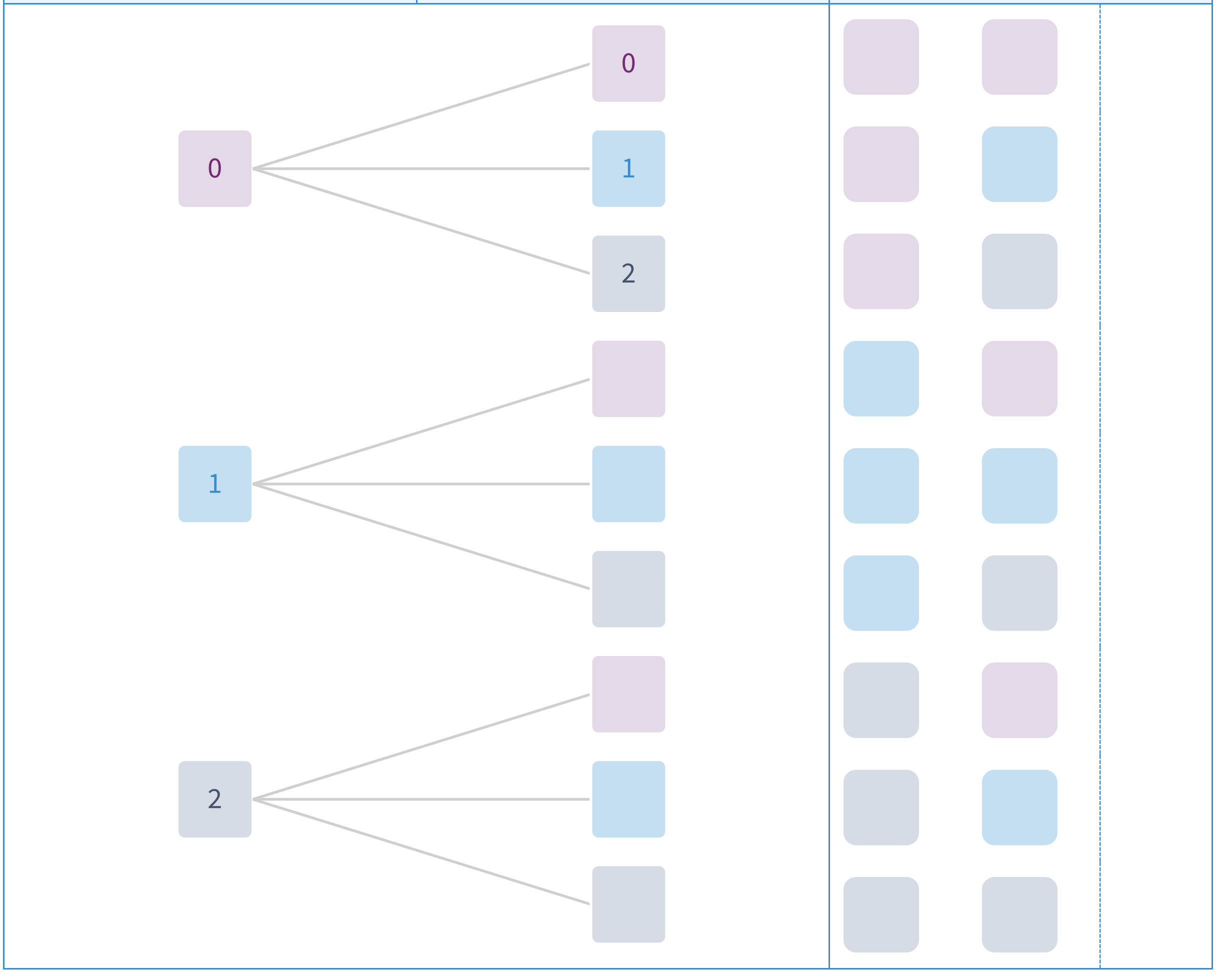
In all, there are _____ different codes.

Obviously, the number of different codes is not enough to encode 40 items. Mrs Stocktout tries another option.

Attempt 2

Mrs Stocktout chooses the first two digits from the set $A = \{0; 1; 2\}$. What codes can she form with these digits? How many different codes are there?

1st digit	2nd digit	Possible codes



In all, there are _____ different codes.

To find out **just how many different codes there are**, you can think like this:

How many possibilities are there for each position in the code?

1st digit	2nd digit
_____ possibilities for the 1st digit	_____ possibilities for the 2nd digit
__________ · __________ = ______	

So, in all, there are ___________ different codes:

Attempt 3

Mrs Stocktout continues her research. Help her to complete the table below:

If	$A = \{3; 4\}$	then it can form		different codes.
If	$A = \{7; 8; 9\}$	then it can form		different codes.
If	$A = \{0; 1; 2; 3\}$	then it can form		different codes.
If	$A = \{4; 5; 6; 7; 8; 9\}$	then it can form		different codes.
If	$A = \{0; 1; 2; 3; 4; 5; 6; 7; 8; 9\}$	then it can form		different codes.

How many different digits must Mrs Stocktout choose to obtain at least 40 different codes?

Give an example for the set A :

Using the digits in the set A defined above, give some examples of the codes generated this way:

M13 A check digit for the new encoding

Mrs Stocktout decided to go for the first two digits in the set $\boldsymbol{A = \{0; 1; 2; 3; 4; 5; 6\}}$.

1st digit	2nd digit
_____ possibilities for the 1st digit	_____ possibilities for the 2nd digit
__________ · __________ = ______	

With these digits, she can form ________ different codes.

To improve the performance of the scans, Mrs Stocktout decided to **add a check digit** at the end of the code. In the end, the new code consisted of 3 digits:

- The first two digits a and b are chosen from the elements of the set A.
- The third digit, the check digit, is obtained as follows:
  - You calculate the sum of the digits : $S = a + b$
  - Carry out the Euclidean division of S by the divisor d (choose one);
  - The check digit will be the remainder of this division.

Let's look at an example:

Example 01: Euclidean division by $d = 10$

The check digit is the remainder of the Euclidean division of the sum of the digits by 10.

Code	Sum of digits S	Check digit	Full code
02	2	2	022
43			
56			
66			
00			

Are there any codes ending in 8? If so, which ones?

Are there any codes ending in 9? If so, which ones?

Let's see if this choice makes it possible to recover damaged codes:

Damaged code	Check digit	Sum of digits	Value that the sum must take	Value of x	Initial code
$x60$	0	$x + 6$	10	4	460
$x50$					
$0x1$					
$6x1$					

What can you conclude? Tick the right answer:

If $d = 10$	the check digit **is used to find the original code**, as there is only **one possibility** for the value of x.	☐
	the check digit **cannot be used to find the initial code**, as there are **several possibilities** for the value of x.	☐

Mrs Stocktout asks the following question: is it possible to **choose a divisor less than 10**?

Example 02: Euclidean division by $d = 2$

The check digit is the remainder of the Euclidean division of the sum of the digits by 2.

Code	Sum of digits S	Check digit	Full code
02	2	0	020
43			
56			
66			
00			

All codes end with ____ .

Let's see if this choice makes it possible to recover damaged codes:

Damaged code	Check digit	Sum of digits	Value that the sum must take	Value of x	Initial code
$x60$	0	$x + 6$	6 or 8 or 10 or 12	0 or 2 or 4 or 6	060 or 260 or 460 or 660
$x50$					
$0x1$					
$6x1$					

What can you conclude? Tick the right answer :

If $d = 2$	the check digit **is used to find the original code**, as there is only **one possibility** for the value of x.	☐
	the check digit **cannot be used to find the initial code**, as there are **several possibilities** for the value of x.	☐

Example 03: Euclidean division by $d = 3$

The check digit is the remainder of the Euclidean division of the sum of the digits by .3

Code	Sum of digits S	Check digit	Full code
02	2	2	022
43			
56			
66			
00			

All codes end with ____ .

Let's see if this choice makes it possible to recover damaged codes:

Damaged code	Check digit	Sum of digits	Value that the sum must take	Value of x	Initial code
$x60$	0	$x + 6$	6 or 9 or 12	0 or 3 or 6	060 or 360 or 660
$x50$					
$0x1$					
$6x1$					

What can you conclude? Tick the right answer :

If $d = 3$	the check digit **is used to find the original code**, as there is only **one possibility** for the value of x.	☐
	the check digit **cannot be used to find the initial code**, as there are **several possibilities** for the value of x.	☐

Example 04: Euclidean division by $d = 4$

The check digit is the remainder of the Euclidean division of the sum of the digits by 4.

Code	Sum of digits S	Check digit	Full code
02	2	2	022
43			
56			
66			
00			

All codes end with ____ .

Let's see if this choice makes it possible to recover damaged codes:

Damaged code	Check digit	Sum of digits	Value that the sum must take	Value of x	Initial code
$x60$	0	$x + 6$	8 or 12	2 or 6	260 or 660
$x50$					
$0x1$					
$6x1$					

What can you conclude? Tick the right answer :

If $d = 4$	the check digit **is used to find the original code**, as there is only **one possibility** for the value of x.	☐
	the check digit **cannot be used to find the initial code**, as there are **several possibilities** for the value of x.	☐

Example 05: Euclidean division by $d = 5$

The check digit is the remainder of the Euclidean division of the sum of the digits by 5.

Code	Sum of digits S	Check digit	Full code
02	2	2	022
43			
56			
66			
00			

All codes end with ____ .

Let's see if this choice makes it possible to recover damaged codes:

Damaged code	Check digit	Sum of digits	Value that the sum must take	Value of x	Initial code
$x60$	0	$x + 6$	10	4	460
$x50$					
$0x1$					
$6x1$					

What can you conclude? Tick the right answer :

If $d = 5$	the check digit **is used to find the original code**, as there is only **one possibility** for the value of x.	☐
	the check digit **cannot be used to find the initial code**, as there are **several possibilities** for the value of x.	☐

Example 06: Euclidean division by $d = 6$

The check digit is the remainder of the Euclidean division of the sum of the digits by 6.

Code	Sum of digits S	Check digit	Full code
02	2	2	022
43			
56			
66			
00			

All codes end with ____ .

Let's see if this choice makes it possible to recover damaged codes:

Damaged code	Check digit	Sum of digits	Value that the sum must take	Value of x	Initial code
$x60$	0	$x + 6$	6 or 12	0 or 6	060 or 660
$x50$					
$0x1$					
$6x1$					

What can you conclude? Tick the right answer :

If $d = 6$	the check digit **is used to find the original code**, as there is only **one possibility** for the value of x.	☐
	the check digit **cannot be used to find the initial code**, as there are **several possibilities** for the value of x.	☐

Example 07: Euclidean division by $d = 7$

The check digit is the remainder of the Euclidean division of the sum of the digits by 7.

Code	Sum of digits S	Check digit	Full code
02	2	2	022
43			
56			
66			
00			

All codes end with ____ .

Let's see if this choice makes it possible to recover damaged codes:

Damaged code	Check digit	Sum of digits	Value that the sum must take	Value of x	Initial code
$x60$	0	$x + 6$	7	1	160
$x50$					
$0x1$					
$6x1$					

What can you conclude? Tick the right answer :

If $d = 7$	the check digit **is used to find the original code**, as there is only **one possibility** for the value of x.	☐
	the check digit **cannot be used to find the initial code**, as there are **several possibilities** for the value of x.	☐

Example 08: Euclidean division by $d = 8$

The check digit is the remainder of the Euclidean division of the sum of the digits by 8.

Code	Sum of digits S	Check digit	Full code
02	2	2	022
43			
56			
66			
00			

All codes end with ____ .

Let's see if this choice makes it possible to recover damaged codes:

Damaged code	Check digit	Sum of digits	Value that the sum must take	Value of x	Initial code
$x60$	0	$x + 6$	8	2	260
$x50$					
$0x1$					
$6x1$					

What can you conclude? Tick the right answer :

If $d = 8$	the check digit **is used to find the original code**, as there is only **one possibility** for the value of x.	☐
	the check digit **cannot be used to find the initial code**, as there are **several possibilities** for the value of x.	☐

Example 09: Euclidean division by $d = 9$

The check digit is the remainder of the Euclidean division of the sum of the digits by 9.

Code	Sum of digits S	Check digit	Full code
02	2	2	022
43			
56			
66			
00			

All codes end with ____ .

Let's see if this choice makes it possible to recover damaged codes:

Damaged code	Check digit	Sum of digits	Value that the sum must take	Value of x	Initial code
$x60$	0	$x + 6$	9	3	360
$x50$					
$0x1$					
$6x1$					

What can you conclude? Tick the right answer :

If $d = 9$	the check digit **is used to find the original code**, as there is only **one possibility** for the value of x.	☐
	the check digit **cannot be used to find the initial code**, as there are **several possibilities** for the value of x.	☐

Summary table

Indicates by ✓ if the check digit allows the initial code to be recovered and by ✗ if it does not.

$$A = \{0\,;1\,;2\,;3\,;4\,;5\,;6\}$$

Euclidean division by $d = \ldots$	The check digit is used to retrieve the initial code.
$d = 2$	
$d = 3$	
$d = 4$	
$d = 5$	
$d = 6$	
$d = 7$	
$d = 8$	
$d = 9$	
$d = 10$	

What do you notice?

M14 An optimal code

Mrs Stocktout notices that the check digit allows her to recover a missing digit in the code from a divisor strictly greater than 6. But why? And why can't she find the original code if the divisor is less than 6? Her scientific mind arouses her curiosity and leads her to investigate this phenomenon in more detail.

Let's recall some facts that were discovered during the previous activities:

- There are families of digits which have the same remainder by Euclidean division by divisor d.
  Example:
  The digits 1,3 and 5 all have a remainder equal to _____ by the Euclidean division by divisor 2.

- The remainder of the Euclidean division by the divisor d of a sum of two numbers is equal to the sum of the remainders of the Euclidean division by these two numbers.
  Example:
  If $d = 4$, the remainder of the Euclidean division of the number 5 is _____ and that of the number 6 is _____. The remainder of the sum $S = 5 + 6 = 11$ is equal to the sum of the remainders __________ .

Mrs Stocktout's codes are made up of :

- two digits a and b selected from ; $A = \{0; 1; 2; 3; 4; 5; 6\}$.
- the check digit, which is equal to the remainder of the Euclidean division of the sum of the digits $S = a + b$ by a divisor d.

We can therefore conclude that :

The **remainder** of the Euclidean division of divisor d **of the sum of the numbers is equal to the remainder** of the Euclidean division of divisor d **of the sum of the remainders** of the numbers a and b.

So **all we have to do is look at the remainders of the different numbers in the set** A by Euclidean division of the divisor d to find the origin of the problem of the missing digits!

	1st digit	2nd digit	Check digit
Code	a	b	r
Remainder of Euclidean division by d	r_1	r_2	The rest of $r_1 + r_2$

Complete the table below and highlight any duplicates in each column.

Elements of the set A	Remainder of Euclidean division by								
	$d = 2$	$d = 3$	$d = 4$	$d = 5$	$d = 6$	$d = 7$	$d = 8$	$d = 9$	$d = 10$
0	0								
1	1								
2	0								
3	1								
4	0								
5	1								
6	0								

From $d =$ _____ onwards, there are no more duplicates in the columns. This means that all the remainders are different, which explains why we are able to find a missing digit in these cases!

Let's look at a few final examples to convince us:

- For $d = 2$, it is not possible to recover the initial code from the check digit, because the codes $1\mathbf{1}0, 1\mathbf{3}0$ and 1__0 all have the same check digit (0). These codes have the same check digit because the numbers $\mathbf{1}, \mathbf{3}$ and _____ all have the same remainder by the Euclidean division of the divisor $d = 2$.
- For $d = 3$, it is not possible to recover the initial code from the check digit, because the codes $6\mathbf{0}0, 6\mathbf{3}0$ and 6__0 all have the same check digit (0). These codes have the same check digit because the numbers $\mathbf{0}, \mathbf{3}$ and ____ all have the same remainder by the Euclidean division of the divisor $d = 3$.
- For $d = 4$, it is not possible to recover the initial code from the check digit, because the codes 3__1 and $3\mathbf{6}1$ all have the same check digit (1). These codes have the same check digit, because the numbers _____ and $\mathbf{6}$ all have the same remainder by the Euclidean division of divisor $d = 4$.
- For $d = 5$, it is not possible to recover the initial code from the check digit, because the codes $0\mathbf{1}1$ and 0__1 all have the same check digit (1). These codes have the same check digit, because the numbers $\mathbf{1}$ and _____ all have the same remainder by the Euclidean division of divisor $d = 5$.
- For $d = 6$, it is not possible to recover the initial code from the check digit, because the codes $4\mathbf{0}4$ and 4__4 all have the same check digit (2). These codes have the same check digit, because the numbers $\mathbf{0}$ and _____ all have the same remainder by the Euclidean division of divisor $d = 6$.
- If $d \geq 7$, then it is possible to recover the initial code from the check digit, because all the numbers have different remainders.

M15 Lessons from lines and digits

We have come across two methods to create barcodes:

- Alice's is based on the sum of the digits.
- Bob's is based on the product of digits.

Which one is the best and why?

Mrs Stocktout has created barcodes where the check digit is obtained by means of Euclidean divisions. If she chooses the first two digits from 0 to 6, which Euclidean divisions can she use to generate her check digit?

To create as many barcodes as possible, which set of digits should you choose?

Combine all the elements and explain an effective method to create 12-digit barcodes with a check digit (13 digits in all).

M16 European Article Number - EAN

To standardise trade in goods, several organisations have established standards for the electronic encoding of products. In Europe, products carry a unique 13-digit EAN (European Article Number) that provides information on the country of origin, the producer and the product. This number is often printed as a barcode to facilitate electronic reading and computer processing of sales and distribution.

Here is an extract from the catalogue of instructions for calculating the check digit of an item encoded by the EAN-13 system.

3 453120 236458

Step															Calculation
1	Assign position	1	2	3	4	5	6	7	8	9	10	11	12	13	
	Code	3	4	5	3	1	2	0	2	3	6	4	5	**?**	
2	Add digits in even positions		4		3		2		2		6		5		22
3	Multiply the result by 3														66
4	Add digits in odd positions	3		5		1		0		3		4			16
5	Add the results of steps **3** and **4**														82
6	Take the remainder of the Euclidean division by 10 of the result from step **5**														2
7	Subtract the result of step **6** from 10														8
	Full code	3	4	5	3	1	2	0	2	3	6	4	5	8	

Important note: If the result of step 7 is 10, write 0.

Exercise 01: Calculating the check digit for an EAN-13 code

Look at the barcode on a LUXLAIT yoghurt and repeat the computation described in the instructions above to find the check digit.

Step															Calculation
1	Assign position	1	2	3	4	5	6	7	8	9	10	11	12	13	
	Code													**?**	
2	Add digits in even positions														
3	Multiply the result by 3														
4	Add digits in odd positions														
5	Add the results of steps **3** and **4**														
6	Take the remainder of the Euclidean division by 10 of the result from step **5**														
7	Subtract the result of step **6** from 10														
	Full code														

Exercise 02: Calculating the check digit for an EAN-13 code

Find an EAN-13 (or ISBN-13) barcode and repeat the computation described in the instructions above to find the check digit.

Step															Calculation
1	Assign position	1	2	3	4	5	6	7	8	9	10	11	12	13	
	Code													**?**	
2	Sum digits in even positions														
3	Multiply the result by 3														
4	Sum digits in odd positions														
5	Add the results of steps **3** and **4**														
6	Take the remainder of the Euclidean division by 10 of the result from step **5**														
7	Subtract the result of step **6** from 10														
	Full code														

M17 Detecting erroneous codes

Decide which of the EAN codes below is correct and which is incorrect. Mark the correct codes with

and the incorrect codes with . Correct erroneous codes by adjusting the check digit.

	1	2	3	4	5	6	7	8	9	10	11	12	13
Code A	4	2	4	2	4	2	4	2	4	2	4	2	0

	1	2	3	4	5	6	7	8	9	10	11	12	13
Code B	1	1	1	1	1	1	1	1	1	1	1	1	1

	1	2	3	4	5	6	7	8	9	10	11	12	13
Code C	1	2	3	1	2	3	1	2	3	1	2	3	1

M18 Calculate a missing digit for an EAN-13 code

For the EAN codes below, find the missing digit in each case:

	1	2	3	4	5	6	7	8	9	10	11	12	13
Code	5	0	0	1		0	1	0	0	0	2	6	2

	1	2	3	4	5	6	7	8	9	10	11	12	13
Code	4	0	0		1	2	3	0	0	0	0	0	1

M19 The secrets of weighted sums?

Alice and **Bob** work in a supermarket and have to scan an item whose barcode has been torn:

6 696689 6 69696

This means they have to enter the code **manually** using the keypad at the checkout:

Alice	**Bob**
 6696689669696	 6696689696696

Who entered the code correctly? **Alice**☐ Bob ☐

___________ made a very common error when transcribing the code: swapping the positions of two consecutive digits.

Let's imagine a system for calculating the check digit without the-multiplication-by-3-step. The instructions would then be as follows (for the sake of simplicity, we are also dropping the last instruction, which asked you to subtract the penultimate result from 10):

Step															Calculation
1	Assign position	1	2	3	4	5	6	7	8	9	10	11	12	13	
	Code														
2	Sum all digits														
3	Take the remainder of the Euclidean division by 10 of the result from step **2**														
	Full code														

First, consider the code entered by **Alice**:

Step															Calculation
1	Assign rank	1	2	3	4	5	6	7	8	9	10	11	12	13	
	Code														
2	Add up all the figures														
3	Take the remainder of the Euclidean division by 10 of the result from step **2**														
	Full code														

Do the same calculation for **Bob**'s erroneous code:

Step															Calculation
1	Assign rank	1	2	3	4	5	6	7	8	9	10	11	12	13	
	Code														
2	Add up all the figures														
3	Take the remainder of the Euclidean division by 10 of the result from step **2**														
	Full code														

Bob's mistake

- is detected. ☐
- is not detected. ☐

Repeat the exercise with **alternating coefficients between 1 and 3, using the real rules of the EAN-13 code.**

First, consider the code entered by **Alice**:

Step															Calculation
1	Assign position	1	2	3	4	5	6	7	8	9	10	11	12	13	
	Code													**?**	
2	Sum digits in even positions														
3	Multiply the result by 3														
4	Sum digits in odd positions														
5	Add the results of steps **3** and **4**														
6	Take the remainder of the Euclidean division by 10 of the result from step **5**														
7	Subtract the result of step **6** from 10														
	Full code														

Do the same calculation for **Bob**'s erroneous code:

Step															Calculation
1	Assign position	1	2	3	4	5	6	7	8	9	10	11	12	13	
	Code													**?**	
2	Sum digits in even positions														
3	Multiply the result by 3														
4	Sum digits in odd positions														
5	Add the results of steps **3** and **4**														
6	Take the remainder of the Euclidean division by 10 of the result from step **5**														

7	Subtract the result of step **6** from 10													
	Full code													

Bob's mistake

is detected. ☐

is not detected. ☐