PIT · PROGRAMME FOR INNOVATIVE TEACHING AND TRAINING

3#Dice-ordered solids

3.1 Didactic commentary

Mathematics is often perceived as an abstract discipline, disconnected from the everyday reality of students' lives outside the school setting. To counter this perception, an increasingly adopted pedagogical approach is to incorporate more 'real world' examples into teaching (Coles, 2016).

To anchor learning in a concrete approach, we have chosen to approach solids and volumes through a creative project. Pupils are invited to design their own die by defining its shape, number of faces and the numbers, symbols or letters that will appear on it. The process continues with digital modelling of the die using *Tinkercad* software, followed by 3D printing in the school's Makerspace. The learning is consolidated by a final presentation of their creation. This approach is fully in line with project-based learning (see Section 3.3 *Teaching materials M2*).

Among the many methods likely to improve student motivation, project-based learning has been frequently cited for several decades. It has become everyday practice in vocational education and higher education (Reverdy, 2013).

During the project, the students construct their die by using their existing knowledge and skills. This approach enables them to build their knowledge through a process of constructive trial and error. The interdisciplinary nature of the project forces them to integrate knowledge from different fields, thereby fostering the development of essential skills such as problem solving, information retrieval, documentation, control, critical thinking, organisation and planning.

This pedagogical approach has the significant advantage of making the students autonomous, positioning them as true authors of their creation. By investing themselves in this project, they take charge of planning the stages of production and ensuring that they see their work through to completion (Reverdy, 2013).

The teacher's role changes in this context, as he or she evolves from teacher to facilitator and motivator. They do not help the students directly, but make sure that they are on the right track and that their project is progressing.

One of the common criticisms of project-based learning is that it takes too long. Firstly, in this module we have ensured that the project covers exactly the elements of the official mathematics curriculum relating to solids and volumes. The final presentation of the die by the pupils forces them to confront the content, knowledge and skills relating to solids and volumes in the curriculum. As project-based learning does not work miracles, we have included mini-lessons (section 3.3. *Teaching materials M3*) relating to these different contents. These mini-lessons explain the content and ask students to complete exercises to check and verify their understanding. In this way, each pupil can progress independently and at their own pace: if a pupil knows how to calculate the volume of their die, they don't need to do the mini-lesson and can work on their calculation straight away. However, research shows that discovery-based learning is only effective when learners receive timely feedback and are offered examples of detailed solutions (Alfieri et al., 2011). This is why we provide detailed solutions for all exercises (section *3.7 Solutions*). In this way, students can consult them independently and self-monitor their learning. These mini-lessons also ensure that the knowledge required to complete the project does not exceed the student's knowledge, a second criticism of project-based learning that we often encounter.

Project-based learning also has many advantages (Reverdy, 2013). In project-based learning, pupils and students learn by being active and maintaining a link with the real world, which enables them to nurture communication, cooperation, creativity and in-depth reflection. Attention to learning processes, not just content, is beneficial (Reverdy, 2013).

Educational researcher Robert DeHaan highlights the importance of promoting creative thinking in natural science education (DeHaan, 2009). According to DeHaan, natural science teaching is still often far from promoting this transferable knowledge in a sustainable way. The teaching is heavily influenced by facts and recipes, leaving learners with little knowledge and generating boredom. Studies have also shown that activities that encourage creativity significantly improve learning success in natural science teaching.

Research into project-based teaching also recommends cooperation between teachers from different disciplines. This is what we suggest in the fifth phase of the project, where the pupils have to invent their own game using the die created in the previous phases. We recommend doing this part of the project with the French or Digital Sciences teacher (see Section *3.4 Interdisciplinary ideas* for more details).

We suggest starting the lesson with dice games organised into learning stations. Pupils explore six different stations in groups, at their own pace, without having to go through them all in the first two sessions.

A recent study (Abdelmalak, 2024) demonstrates the effectiveness of this model in developing pupils' analytical, generative and evaluative skills. These cognitive improvements are probably the result of a combination of factors: involvement in stimulating tasks, collaboration between peers, alternation between individual and group learning, pedagogical support and a favourable learning environment. As several researchers have pointed out, intellectually stimulating activities contribute significantly to the development of fundamental mathematical skills (Abdelmalak, 2024).

At each station, students document their experience and answer reflective questions in writing. Dionne Cross (2009) points out that "writing is a powerful strategy for encouraging learning", offering greater benefits than simple oral argumentation. This effectiveness is explained by metacognitive activation: by formulating their thoughts in writing, students diagnose the problem, plan their approach and constantly question their reasoning.

These games also introduce probability, a field with well-established educational challenges. The use of dice represents an accessible approach to introducing the concept of equiprobability. As Capaldi (2021) notes, discovery-based learning through a variety of games stimulates motivation and develops analytical reasoning, making up for teachers' difficulty in creating engaging problems that illustrate mathematical applications in concrete terms. The questions proposed after each game focus on developing intuition rather than formal mastery of probability theory.

By the end of the module, students will have developed an intuition for probability while assimilating the essential concepts of solids and volumes through games and by designing their own die.

References

1. Abdelmalak, M. M. M. 2024. Promoting selected core thinking skills using math stations rotation. *Research in Mathematics Education*, 1-22. https://doi.org/10.1080/14794802.2024.2344209
2. Alfieri, L., Brooks, P. J., Aldrich, N. J. & Tenenbaum, H. R. 2011. Does Discovery-Based Instruction Enhance Learning? *Journal of Educational Psychology*, 103 (1), 1-18. doi: 10.1037/a0021017.
3. Capaldi, Mindy. 2021. *Teaching Mathematics Through Games*. 1st ed. Providence: American Mathematical Society.
4. Coles, Alf. 2016. *Engaging in Mathematics in the Classroom: Symbols and Experiences*. London ; Routledge.
5. Cross, D. I. 2009. Creating Optimal Mathematics Learning Environments: Combining Argumentation and Writing to Enhance Achievement. *International Journal of Science and Mathematics Education* (2009) 7: 905- 930.
6. DeHaan R. L. 2009. Teaching creativity and inventive problem solving in science. *CBE Life Sci Educ*. Fall;8(3):172-81. doi: 10.1187/cbe.08-12-0081.
7. Reverdy, C. 2013. Des projets pour mieux apprendre ? Dossier d'actualité veille et analyses n° 82 February 2013. https://veille-et-analyses.ens-lyon.fr/DA-Veille/82-fevrier-2013.pdf?v=1361180601

3.2 Lesson planning

01 Conditions of unit

Target group: 7 – 5

Place: A classroom

Materials required: 100 normal dice, a few special dice, tablets or computers, Makerspace

Duration: 4-5 teaching hours

02 Targeted skills

Content :

- Solids and volumes
- Basic notions of probability

Knowledge and skills

The pupil knows

- vocabulary of solids: polyhedron, vertex, edge, face
- common solids: cube, rectangular parallelepiped, prism, right prism, pyramid, cone, sphere, cylinder
- formulae for calculating the volume of a cube, rectangular parallelepiped, right prism, cylinder and pyramid
- formulae for calculating the lateral surface area and total surface area of a cube, rectangular parallelepiped, right prism and cylinder
- the units of length, area and volume.

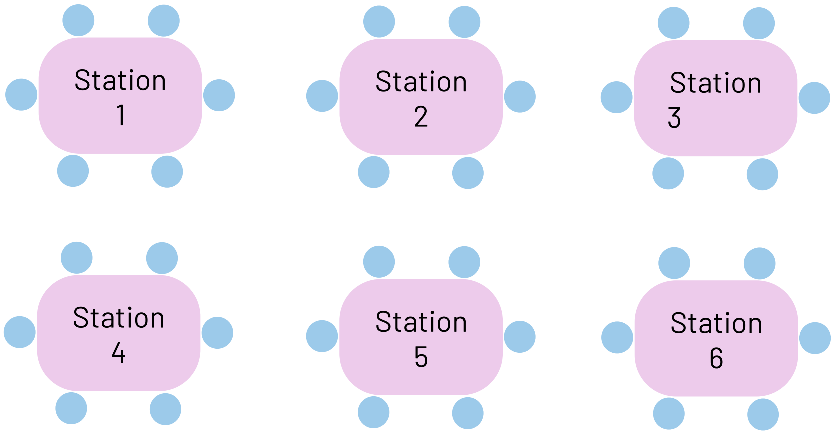
The pupil is able to

- recognise and classify solids
- count the edges, vertices and faces of a solid
- associate a solid with its net
- calculate the volume, lateral area and total area of a solid
- use dynamic geometry software to visualise solids and make their nets.

03 Over the course of the lesson

Setup (before the lesson)

Benches and chairs are arranged in the classroom to form islands for 2 to 5 students.



First two hours of teaching

The first two hours are taught in a ***learning stations*** format. A set of dice is placed on each of the 6 tables. The 6 games are available under *3.3 Teaching materials M1*. All these games are played in groups of 2 to 5 players. To optimise learning by stations, it is preferable for each group to have a similar size. The pupils can form their groups themselves, but the groups will probably be more diverse if the teacher determines their composition in advance. Students should remain in the same group until the end of the learning stations session.

It is advisable to leave at least one of the six stations unoccupied at the start of the activity. This way, the first group to finish can move straight to it. This way, the students don't have to wait to start a new station. In principle, each group spends between 15 and 30 minutes per station. It is not compulsory for all the pupils to try out all the games. The session lasts 2 teaching hours and, depending on the pace of the pupils, a variable number of games will be explored.

The lesson begins with a short introduction (5 minutes), during which the methods of learning stations are explained. The teacher explains to the pupils that at each station they must **read** the rules of the game and **play** a game according to the rules. During the various games, the pupils **use** dice of different shapes and **develop** an intuition for the probabilistic results in relation to the throw of the dice. They also **develop** a strategy to win the game or maximise their winnings.

The number of each station is clearly indicated and each station is equipped with the necessary material for the corresponding game (M1). Most of the games use standard D6 dice. For the best experience, we recommend that you use around a hundred D6 dice for this sequence. The roll of the dice can also be simulated virtually at https://g.co/kgs/EFqAMnE or www.mathigon.org.

Roll the dice

https://g.co/kgs/EFqAMnE

Each student receives handouts (Station 1 to Station 6 in M1) describing the different stations and the considerations to be made. The students complete these documents as they go along, based on their experiences at each station.

The various games can of course be replaced by other dice games. Here is a non-exhaustive list of other games:

- Würfelkönig
- King of Tokyo
- Escape: The Curse of the Temple
- Sagrada
- Tenzi
- Yahtzee
- Pikomino
- Perudo
- Can't stop

Third and fourth hour of teaching

The next two hours are organised around *project-based learning*. The aim is for each pupil to create their own personalised die and possibly invent a new game.

The session begins with a brief introduction (5 minutes) in which the teacher clearly explains the project to the whole class.

The pupils then embark on the work, which is structured into 4 compulsory phases and 1 optional phase:

First phase. The pupils define the characteristics of their personalised die through structured brainstorming. They use the idea-gathering sheets (M2) to organise and clarify their concepts. During this phase, they will encounter potentially new geometric concepts. To support them, 8 mini-lessons (M3) are available on request:

1. Tinkercad tutorials
2. Vocabulary of solids
3. Right prisms

4. Area of a right prism
5. Volume of a right prism
6. Right cylinders
7. Area and volume of a cylinder
8. Dice and probabilities

The idea is not for the teacher to teach these mini-lessons in a classroom setting, but for the students to use them according to their specific needs. Each mini-lesson includes self-assessment exercises. To encourage autonomy, we recommend that the answers (*3.7 Solutions*) be made available in different (more or less controlled) ways: either on the teacher's desk for consultation after completing the exercises, or directly accessible to the students (paper or digital version). The teacher may also decide to run a mini-lesson as a whole class session if the situation so requires.

Second phase. The students create their 3D die using Tinkercad software. A tutorial is available in mini-lesson 1 (to be consulted digitally, as it consists of GIF animations). The students must save their model. It will be used in two different ways in the third and fourth phases.

Third phase. The students create their die, either by 3D printing or on paper (Pepakura). The assistance of the MakerSpace manager is recommended for this stage. Printing is based on the 3D model created in phase 2.

Fourth phase. Pupils use their creation in one of the following ways:

1. Creation of a poster presenting the die with the information detailed in the sheet for phase 4 (M2).
2. Creation of a card for a "Top Trumps" game[1] . Each pupil produces a card describing their die. All the cards in the class make up a complete game, the rules of which are explained in M2.
3. Production of a video in which one of the characteristics of the die is determined experimentally. Four ideas are given in M2. The students will start by researching these ideas in order to fully understand the concepts and how they can be applied concretely to the study of a die. An important point to emphasise concerns the verification of Euler's formula: to obtain convincing results, it is preferable to test this formula on several dice of different shapes. Pupils who choose this approach will have to borrow their classmates' dice to enrich their experimentation. In addition, students are perfectly free to propose and develop their own ideas for experiments beyond those suggested.

   Note: *Section 5.3 M4* of the *PITT VideoMATHon* module gives very precise instructions on how to make a video using a smartphone.

Fifth phase. The fifth phase is optional. Once the personalised die has been created, the work can be continued by creating a game to be played with this die. There are a number of different

[1] Top Trumps is a card game first published in 1978. Each card contains a list of numerical data, and the aim of the game is to compare these values in an attempt to beat and win an opponent's card. https://en.wikipedia.org/wiki/Top_Trumps

forms available on the Internet to help students create games. This phase could also be done in collaboration with the French or Digital Sciences class (see *3.4 Interdisciplinary ideas*).

04 Differentiation possibilities

During the first two hours, the groups progress at their own pace between the different stations (M1). By keeping a station free from the start, students do not need to complete their activities simultaneously. What's more, they don't have to explore all the stations. These two methods ensure that the learning process respects the individual pace of each group of students.

During the next two hours of teaching, the mini-lessons enable effective internal differentiation. As well as optimising classroom time management, mini-lessons (M3) are a powerful tool for differentiating teaching. Students navigate with ease between their personal work, learning stations, available resources and mini-lessons, according to their specific needs. Fully engaged in their learning journey, they develop their autonomy while the teacher adapts his or her support without imposing a uniform pace. As each pupil has different levels of attainment, mini-lessons can either be offered as a resource or made compulsory only for those who will really benefit from them.

05 Other criteria to be met as part of the series of units

a) **Luxembourg context**: This module requires the availability of tablets in class as well as access to a Makerspace within the school. Given the excellent digital infrastructure of Luxembourg schools, this module is particularly suitable for implementation in Luxembourg.

b) **Differentiation**: As indicated in the previous paragraph, the very structure of the module naturally incorporates elements of pedagogical differentiation.

c) **Media Compass**: Competences targeted by the Media Compass[2] :

- Competence 2 - Communication and collaboration: 2.1 Working with others
- Competence 3 - Creating content: 3.3. Modelling, structuring and coding

d) **4C model**: communication, collaboration, creativity, critical thinking: The 4Cs are integrated into this module. During the first two hours of teaching, students are required to collaborate and communicate to play the games and answer the questions on the worksheets. The design of the personalised die is a truly creative process. What's more, the pupils are involved in an autonomous learning process where they must decide for themselves when and which mini-lesson to use to make progress on their project. This empowerment significantly develops their critical thinking skills.

e) **Link with mathematics research:** The solids presented in this module represent special cases of polytopes. Polytopes, particularly in dimensions greater than 2 and 3, are a dynamic and current area of research in mathematics. See also Section *3.5 Further Reading*.

[2] https://edumedia.lu/uebersicht/

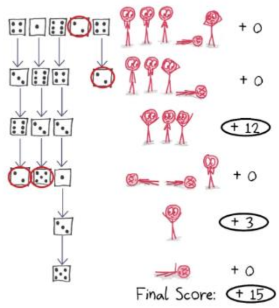
Detailed planning of the lesson

Focus	Social forms / Methods	Materials / Media	Learning process
Lessons 1-2			
Experimenting with dice Exploring probabilities	Learning stations	M1 Paper and pencil Dice of different shapes	Pupils... ...know how to calculate the probability of any specific outcome when rolling a balanced die. ...are familiar with the concept of probability.
Lesson 3-4			
Creating a die	Project-based learning	M2 & M3 Tablets or computer Paper and pencil Makerspace	Pupils... ...know the vocabulary of solids: polyhedron, vertex, edge, face. ...can count the edges, vertices and faces of a solid. ...are able to calculate the volume, lateral area and total area of a solid. ...are able to use dynamic geometry software to visualise solids and make their nets.
Lesson 5 (optional)			
Inventing a game Drawing up the rules of the game	Individual work	M2 Paper and pencil Tablets	Pupils... ...are able to design games of their own creation. ...are able to write rules faithfully reproducing the writing style characteristic of game instructions.

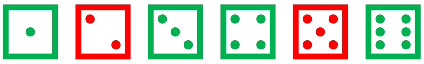
3.3 Teaching materials

M1 Dicey challenge-stations

Station 1: Avoid 2s and 5s![3]



Description of the game



Materials	5 six-sided dice (D6) per player
Number of players	2-5 players
Aim of the game	To get the highest score.
Duration of the game	25 minutes
Rules of the game	Each player rolls all five dice at once. There are two possible outcomes: ▪ All the dice that show a **2** or a **5** are **removed from** the game and the player scores **0 points** for that round. ▪ If no dice show **a 2 or a 5**, then the player increases their score by noting **the sum of the points** obtained. The player takes the remaining dice and throws them again. The same rule applies for each round. The game ends if there are no more dice to throw. The player with **the highest score** wins the game.

[3] This game is *Drop Dead* by Ben Orling (Math with bad drawings), https://mathwithbaddrawings.com/wp-content/uploads/2020/11/Game-28-Drop-Dead.pdf

Example

	Player 1	Score	Dice out of play
1st turn	⚀ ⚀ ⚄ ⚅ ⚅	0	
2nd turn	⚂ ⚃ ⚀ ⚂	0+11=11	⚄
3th turn	⚃ ⚄ ⚁ ⚅	11+ 0=11	⚄
4th turn	⚃ ⚁	11+0=11	⚄ ⚄ ⚁
5th turn	⚅	11+6=17	⚄ ⚄ ⚁ ⚁
6th turn	⚀	17+1=18	⚄ ⚄ ⚁ ⚁
7th turn	⚄	18+0=18	⚄ ⚄ ⚁ ⚁ ⚄
Game over. Final score: 18 points			

Now it's your turn!

Record your results in the table below.

	Scores				
Name of player					
1st round					
2nd round					
3th round					
4th round					
5th round					
6th round					
7th round					
8th round					
9th round					
10th round					
11th round					
12th round					
13th round					
14th round					
15th round					
Winner :					

Reflection after the game

True ✅ or false ❌ :

	✅ ❌	Justification
If you throw six dice in round 1, you will get all the numbers from 1 to 6.		
If you double the number of dice, the score will double.		
It is impossible to get a final score of 0 points.		
One player claims to have played this game with 10 dice for 100 rounds. Is this story plausible?		

Now play **a single game with 20 dice**. Can you score more points than the winner of the previous game?

Game with **20 dice**	Number of dice eliminated (all dice showing 2 or 5)	Score
1st round		
2nd round		
3rd round		
4th round		
5th round		
6th round		
7th round		
8th round		
9th round		
10th round		
11th round		
12th round		
13th round		
14th round		
15th round		

Yes ☐ No ☐

If you increase the number of dice considerably (imagine a game with 1000 dice!), can you get more points from this game?

Station 2: 1=0, when do you stop?[4]

Description of the game

⚀ = 0

Equipment	One 6-sided die (D6) per player
Number of players	2-5 players
Aim of the game	To get the highest score.
Duration of the game	15 minutes
Rules of the game	Each player rolls his or her die. If the result of the die is not 1, the player can write down the score and decide to continue the game. Any results other than 1 can be added to the previous score. If the player's die shows 1, the score is reset to 0 and the game ends. After each throw, the player can decide to stop the game. The player with **the highest score** wins the game.

[4] We created this game ourselves, with a little help from artificial intelligence.

Example

	Player 1	Score	Player 2	Score
1st round	⚃	4	⚄	5
2nd round	⚂	4+3=7	⚂	5+3=8
3rd round	⚅	7+6=13	⚁	8+2=10
4th round	STOP	13	⚄	10+5=15
5th round	/	13	⚀	0
Player 1 wins the game with 13 points against 0 points for player 2.				

Now it's your turn!

Record your results in the table below.

	Scores				
Player's name					
1st round					
2nd round					
3rd round					
4th round					
5th round					
6th round					
7th round					
8th round					
9th round					
10th round					
11th round					
12th round					
13th round					
14th round					
15th round					
Winner :					

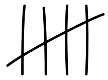
Reflection after the game

True ✅ or false ❌ :

	✅ ❌	Justification
The game definitely ends after the 6thround.		
The maximum score for this game is 36=6x6 points.		
After each result other than 1, the probability of obtaining 1 in the next throw increases.		

Station 3: 3: All the same![5]

Description of the game



⚃ ⚃ ⚃ ⚃ ⚃

Equipment	Five 6-sided dice (D6) per player
Number of players	2-5 players
Aim of the game	All the dice must show the same number.
Duration of the game	20 minutes
Rules of the game	All players roll their dice. On each turn, the players choose which dice they want to put in their reserve. They then re-roll the remaining dice. They are allowed to take the dice out of the reserve and roll them again. The player stops as soon as all the dice show the same result. The player with the **lowest number of turns** wins the game.

Example

	Player 1	Reserve P1	Player 2	Reserve P2
1st round	⚃ ⚂ ⚄ ⚂ ⚁		⚁ ⚄ ⚂ ⚀ ⚅	
2nd round	⚀ ⚂ ⚅	⚂ ⚂	⚁ ⚄ ⚄ ⚀ ⚅	⚄ ⚄
3rd round	⚁ ⚁	⚂ ⚂ ⚂	⚁ ⚂ ⚂	⚄ ⚄
4th round	⚁ ⚄	⚂ ⚂ ⚂	⚀ ⚅ ⚁	⚄ ⚄
5th round	⚂ ⚃	⚂ ⚂ ⚂	⚀ ⚀ ⚀	⚄ ⚄
6th round	⚀	⚂ ⚂ ⚂ ⚂	⚅ ⚁	⚀ ⚀ ⚀
7th round	⚅	⚂ ⚂ ⚂ ⚂	⚄ ⚂	⚀ ⚀ ⚀
8th round	⚁	⚂ ⚂ ⚂ ⚂	⚂ ⚁	⚀ ⚀ ⚀
9th round	⚃	⚂ ⚂ ⚂ ⚂	⚀ ⚀	⚀ ⚀ ⚀
10th round	⚂	⚂ ⚂ ⚂ ⚂	STOP	⚀ ⚀ ⚀ ⚀ ⚀
	STOP	⚂ ⚂ ⚂ ⚂ ⚂		
Player 2 wins, because they were able to stop his game on turn 9th.				

[5] We created this game ourselves, with a little help from artificial intelligence.

Now it's your turn!

Write down the number of rounds in the table below. Use vertical lines to count your turns

Player's name					
Number of rounds					
Winner :					

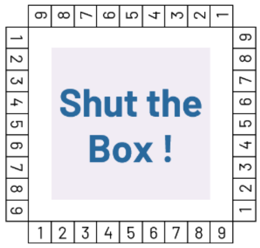
Reflection after the game

True ✅ or false ❌ :

	✅ ❌	Justification
It is impossible to stop the game after the first round.		
If you roll the same die 6 times, you can be sure that it will show the face with the desired result at least once.		
It is more advantageous to collect even numbers than odd numbers.		

Station 4: Shut the box! [6]

Game description



Materials	Two 6-sided dice (D6) per player
Number of players	2-5 players
Aim of the game	To score the fewest penalty points. Be the last player to reach 45 penalty points (or more) to win the game.
Duration of the game	30 minutes
Rules of the game	The youngest player starts the game. They throw the two dice, and there are two possible outcomes: 1. **They can close at least one box**: A box can be closed if one of the dice shows the number of an open box or if the sum of the two dice corresponds to the number of an open box. 2. **They cannot close any box**: In this case, his turn ends and the next player takes his turn. If the player manages to close at least one box, they throw the dice again and continues his turn until they can no longer play (i.e. they fall into the second scenario). The round ends as soon as the last player can no longer continue his turn. At the end of each round, each player counts his penalty points, which correspond to the sum of the numbers of the boxes left open. Rounds continue until all players, except one, reach a penalty score of 45 points or more (45 being the sum of all the boxes, as 1 + 2 + … + 9 = 45). Penalty points accumulate as the game progresses, gradually eliminating players until only one remains, who wins the game.

[6] This game corresponds to the popular game *Shut the box*, the origins of which remain uncertain.

Example Here is an example of a round with 4 players.

Player 1	Closed squares	Player 2	Closed squares	Player 3	Closed squares	Player 4	Closed squares
⚄ ⚂	1 2 ~~3~~ 4 ~~5~~ 6 7 8 9	⚁ ⚁	1 2 3 ~~4~~ 5 6 7 8 9	⚃ ⚁	1 ~~2~~ 3 ~~4~~ 5 6 7 8 9	⚅ ⚄	1 2 3 4 ~~5~~ ~~6~~ 7 8 9
⚃ ⚄	1 2 ~~3~~ 4 ~~5~~ 6 7 8 ~~9~~	⚂ ⚅	1 2 ~~3~~ ~~4~~ 5 ~~6~~ 7 8 9	⚅ ⚅	1 ~~2~~ 3 ~~4~~ 5 ~~6~~ 7 8 9	⚃ ⚂	1 2 3 4 ~~5~~ ~~6~~ ~~7~~ 8 9
⚀ ⚅	~~1~~ 2 ~~3~~ 4 ~~5~~ ~~6~~ 7 8 ~~9~~	⚃ ⚀	1 2 ~~3~~ ~~4~~ ~~5~~ ~~6~~ 7 8 9	⚂ ⚂	1 ~~2~~ ~~3~~ ~~4~~ 5 ~~6~~ 7 8 9	⚂ ⚃	1 2 ~~3~~ ~~4~~ ~~5~~ ~~6~~ ~~7~~ 8 9
⚄ ⚅	STOP	⚄ ⚂	1 2 ~~3~~ ~~4~~ ~~5~~ ~~6~~ 7 ~~8~~ 9	⚀ ⚁	~~1~~ ~~2~~ ~~3~~ ~~4~~ 5 ~~6~~ 7 8 9	⚃ ⚅	STOP
		⚃ ⚃	STOP	⚁ ⚃	STOP		
Penalty points : $2+4+7+8=21$		Penalty points : $1+2+7+9=19$		Penalty points : $5+7+8+9=29$		Penalty points : $1+2+8+9=20$	

It's your turn!

Record your results in the table below.

1st round		
Player's name	Boxes	Penalty points
	1 2 3 4 5 6 7 8 9	
	1 2 3 4 5 6 7 8 9	
	1 2 3 4 5 6 7 8 9	
	1 2 3 4 5 6 7 8 9	
	1 2 3 4 5 6 7 8 9	

2nd round		
Player's name	Boxes	Penalty points
	1 2 3 4 5 6 7 8 9	
	1 2 3 4 5 6 7 8 9	
	1 2 3 4 5 6 7 8 9	
	1 2 3 4 5 6 7 8 9	
	1 2 3 4 5 6 7 8 9	

3e round		
Player's name	Boxes	Penalty points
	1 2 3 4 5 6 7 8 9	
	1 2 3 4 5 6 7 8 9	
	1 2 3 4 5 6 7 8 9	
	1 2 3 4 5 6 7 8 9	
	1 2 3 4 5 6 7 8 9	

4th round		
Player's name	Boxes	Penalty points
	1 2 3 4 5 6 7 8 9	
	1 2 3 4 5 6 7 8 9	
	1 2 3 4 5 6 7 8 9	
	1 2 3 4 5 6 7 8 9	
	1 2 3 4 5 6 7 8 9	

5th round		
Player's name	Boxes	Penalty points
	1 2 3 4 5 6 7 8 9	
	1 2 3 4 5 6 7 8 9	
	1 2 3 4 5 6 7 8 9	
	1 2 3 4 5 6 7 8 9	
	1 2 3 4 5 6 7 8 9	

6th round		
Player's name	Boxes	Penalty points
	1 2 3 4 5 6 7 8 9	
	1 2 3 4 5 6 7 8 9	
	1 2 3 4 5 6 7 8 9	
	1 2 3 4 5 6 7 8 9	
	1 2 3 4 5 6 7 8 9	

7th round		
Player's name	Boxes	Penalty points
	1 2 3 4 5 6 7 8 9	
	1 2 3 4 5 6 7 8 9	
	1 2 3 4 5 6 7 8 9	
	1 2 3 4 5 6 7 8 9	
	1 2 3 4 5 6 7 8 9	

8th round		
Player's name	Boxes	Penalty points
	1 2 3 4 5 6 7 8 9	
	1 2 3 4 5 6 7 8 9	
	1 2 3 4 5 6 7 8 9	
	1 2 3 4 5 6 7 8 9	
	1 2 3 4 5 6 7 8 9	

Reflection after the game

True ✓ or false ✗ :

	✓ ✗	Justification
It is possible for a player's round to end at the 2nd round.		
It is not possible to get 45 penalty points in the 1st round.		
With 2 dice, a sum of 7 is the most likely result.		

Station 5: Farkle![7]

Game description

Tutorial:

How to Play Farkle

https://www.youtube.com/watch?v=EvWcUDYB9wQ

Materials	Six 6-sided dice (D6) per player
Number of players	2-5 players
Aim of the game	To be the first to reach 5,000 points by rolling 6 dice and forming winning combinations.
Duration of the game	30 minutes
Rules of the game	The aim of the game is to be the first player to reach **5,000 points** by rolling **6 dice** and forming winning combinations. The order of play is decided by a **roll of the dice**: each player throws a die, and the player with the highest number goes first. If there is a tie, the players concerned roll again until the first player is chosen. 1 - First round: rolling the dice and scoring points 1A: Roll the 6 dice At the start of each round, a player rolls **all 6 dice**: • **If at least one of the dice scores points**: the player chooses between **cashing in** or **rolling again** to try to improve their score. • **If none of the dice score points**: this is a **Farkle**, and the player loses all his points for that turn. The player's turn ends immediately. 1B: Reroll the dice to improve your score

[7] Farkle (or Farkel) is a dice game comparable to 1000/5000/10000, Cosmic Wimpout, Greed, Hot Dice, Squelch, Zilch, or Zonk. Although its roots as a popular form of entertainment are unknown, the game has been around since at least the mid-1980s. It has been marketed since 1996 under the name "Pocket Farkel" by Legendary Games Inc. Although the basic principles of the game are clearly defined, there is nevertheless a wide variety of variations, both in the methods of play and in the scoring systems.

The player may re-roll as long as they set aside **at least one winning die for each roll**. You do not have to keep all the dice that score points, but you must keep at least one for each re-roll.

1C: Collecting the points

After each throw, the player can choose **to stop and add up his score**. As long as the points are not cashed in, they remain **at risk of being lost in** the event of a Farkle.

1D: End of turn

The round ends when :

- The player **gets a Farkle** (no winning dice). In this case, no points are scored.
- The player decides **to cash in his points** after a throw.

1E: Bonus throw

If a player manages to score with **all 6 dice**, they get a **bonus throw** and can start the process again. However, if they make a **Farkle** during their bonus throw, they **lose all the points they have earned** during this round, including those for the first few throws.

1F: Calculating the score

Dice that have been set aside **cannot be combined with newly rolled dice** to create new combinations. The score is calculated according to the following rules:

Combination	Points
5	50 points
1	100 points
3 identical dice (2-6)	Dice value × 100 (e.g.: 3 dice with value 2 gives 200, 3 dice with value 3 gives 300)
3 identical dice (1)	1,000 points
4 identical dice	1,000 points
5 identical dice	2,000 points
6 identical dice	3,000 points
Suite (1-2-3-4-5-6)	1,500 points
3 pairs	1,500 points
4 identical dice + 1 pair	1,500 points
Twice 3 identical dice	2,500 points

2 - The turn passes to the next player

Play continues **clockwise** (or alternately if there are only 2 players).

3 - Continue to 5,000 points and declare a winner

Players take turns playing **until one player reaches 5,000 points**. At that point, the other players have **one last chance to beat his score**. The player with the most points at the end of this final round is declared **the winner**.

Example Here is an example of a 2-player round.

	Player 1	Winning dice	Set aside dice	Points P1
1st round	⚀ ⚂ ⚁ ⚄ ⚂ ⚁	⚀ ⚄	⚀	100
2nd round	⚁ ⚃ ⚁ ⚁ ⚅	⚁ ⚁ ⚁ ⚄	⚁ ⚁ ⚁	200
3rd round	⚄ ⚅	⚄	⚄	50
	STOP		Total points earned:	350

	Player 1	Winning dice	Set aside dice	Points P2
1st round	⚀ ⚂ ⚁ ⚄ ⚃ ⚅	⚀ ⚁ ⚂ ⚃ ⚄ ⚅	⚀ ⚁ ⚂ ⚃ ⚄ ⚅	1500
2nd round	⚂ ⚄ ⚄ ⚄ ⚅ ⚅	⚄ ⚄ ⚄	⚄ ⚄ ⚄	500
3rd round	⚅ ⚂ ⚁	none	none	
	FARKLE!		Total points earned:	0

Now it's your turn!

Record your results in the table below. After each round, add the points from the previous round.

	Points				
Player's name					
1st round					
2nd round					
3rd round					
4th round					
5th round					
6th round					
7th round					
8th round					
9th round					
10th round					
11th round					
12th round					
13th round					
14th round					
15th round					
Winner :					

Reflection after the game

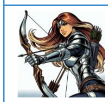
True ✅ or false ❌ :

	✅ ❌	Justification
If you throw all six dice at the start, it is impossible to get a Farkle.		
With a single die, there is a 50% chance of getting 1 or 5.		
With two dice, the probability of getting 1 at least once is $\frac{11}{36} \approx 31\%$.		

Station 6: Lord of the Dice

Game description

Materials	▪ Three 4-sided dice (D4) ▪ Five 6-sided dice (D6) ▪ Four 8-sided dice (D8) ▪ Two 10-sided dice (D10) ▪ Two 12-sided dice (D12)
Number of players	2-5 players
Aim of the game	Defeat all the enemies to collect the magic potion.
Duration of the game	20 minutes
Rules of the game	Each player takes on the role of one of the 5 heroes: the warrior, the magician, the archer, the dwarf and the druid. If there are fewer than 5 players, some players will have to play several heroes at the same time. Check out the list below to discover the special characteristics of the different heroes.

Heroes	Category	Dice	Special feature
	Warrior	5 D6	The warrior's total attack strength is obtained by adding up the results of his dice. The warrior decides how many dice to use to calculate his attack strength.
	Magician	3 D4	The magician can use the result of one of his dice to multiply the attack strength of one of his comrades by the result of this die.
	Archer	4 D8	The archer's total attack strength is obtained by adding up the results of his dice. The archer decides how many dice to use to calculate his attack strength.
	Druid	2 D10	If one of the dice shows a result of 7 or more, the druid can divide the enemy's attack strength by 2.
	Dwarf	2 D12	The dwarf's total attack strength is obtained by choosing the highest value of his dice.

Each hero has a collection of dice, the results of which are used to calculate their collective attack strength or to reduce the enemy's attack strength.

	For each round, the players roll all their currently available dice. They then decide together which dice will be used to calculate their attack strength. If the attack strength of the heroes exceeds that of the enemy, then the heroes win and can attack the next enemy. All the dice involved in the battle are put on the game board and are now out of the game. If the dice results do not defeat the enemy, the heroes can re-roll all their dice a second time. If, after this second attempt, the heroes still fail to defeat the enemy, the heroes lose the game. A hero without dice can no longer take part in the battle. Heroes win the game if they manage to defeat the last enemy.
Example	▪ The warrior rolls 5 D6 dice. Result: 1, 1, 2, 4, 5 ▪ The magician rolls 3 D4 dice. Result: 1, 2, 2 ▪ The dwarf rolls 2 D12 dice. Result: 6, 10 ▪ The archer and the druid no longer have dice. ▪ The enemy has an attack strength of 30. The dwarf's attack strength is 10. The magician can double the dwarf's strength with his result of 2. The warrior invests three dice (2, 4 and 5) to obtain an attack strength of 11. Calculation : $2 \cdot 10 + (2 + 4 + 5) = 20 + 11 = 31$ Together, the heroes have an attack strength of 31, which is greater than that of the enemy. The heroes win. The dice used (1 D12, 1 D4 and 3 D6) are placed on the board in the line of the enemy in question. The warrior now only has 2 dice for the next turn. The magician has 2 dice and the dwarf now has 1 die for the next turn.

It's up to you!

Assign your roles and get your dice! Read over the special characteristics of the different heroes carefully.

Heroes	Category	Dice	Special feature	Player's name
	Warrior	5 D6	The warrior's total attack strength is obtained by adding up the results of his dice. The warrior decides how many dice to use to calculate his attack strength.	
	Magician	3 D4	The magician can use the result of one of his dice to multiply the attack strength of one of his comrades by the result of this die.	
	Archer	4 D8	The archer's total attack strength is obtained by adding up the results of his dice. The archer decides how many dice to use to calculate his attack strength.	
	Druid	2 D10	If one of the dice shows a result of 7 or more, the druid can divide the enemy's attack strength by 2.	
	Dwarf	2 D12	The dwarf's total attack strength is obtained by choosing the highest value of his dice.	

The time has come for battle! If you're ready, turn the page and confront your enemies!

Fight enemy after enemy to collect the magic potion!

Challenge	Enemy	Enemy attack force	Dice used	Hero attack strength	Victory? ✓ ✗
1		4			
2		8			
3		12			
4		20			
5		30			
6		50			
End					

M2 Die-sign Your Own!

It's your turn: Create your own **customised die** and **invent a game** using it!

Phase 1: Gathering ideas

To find **ideas**, try answering the following questions?

<table>
<tr><td>What shape will your die be? Try making a sketch!</td><td>Try to make a net of the solid you've sketched!</td></tr>
<tr><td colspan="2">How big will your die be?</td></tr>
<tr><td colspan="2">How many faces will your die have?</td></tr>
<tr><td colspan="2">How many edges will your die have?</td></tr>
</table>

How many **vertices** will your die have?
Will all the faces be identical? Specify **the geometric shape of** the faces.
What **numbers** or **letters** will be on the faces of your die?
How many dice will you need?
Does the colour of the dice play a role?
What **game** do you want to play with your dice?

3D modelling

Log on to www.tinkercad.com and create your personal account.

Then you can start modelling your own dice using the platform's features.

 …

[See mini lesson 1 for tutorials on using Tinkercad].

Phase 3: 3D printing (filament or resin)

Ask your school's Makerspace manager to print your solid in 3D or on paper (Pepakura).

Phase 4: Presentation of your die

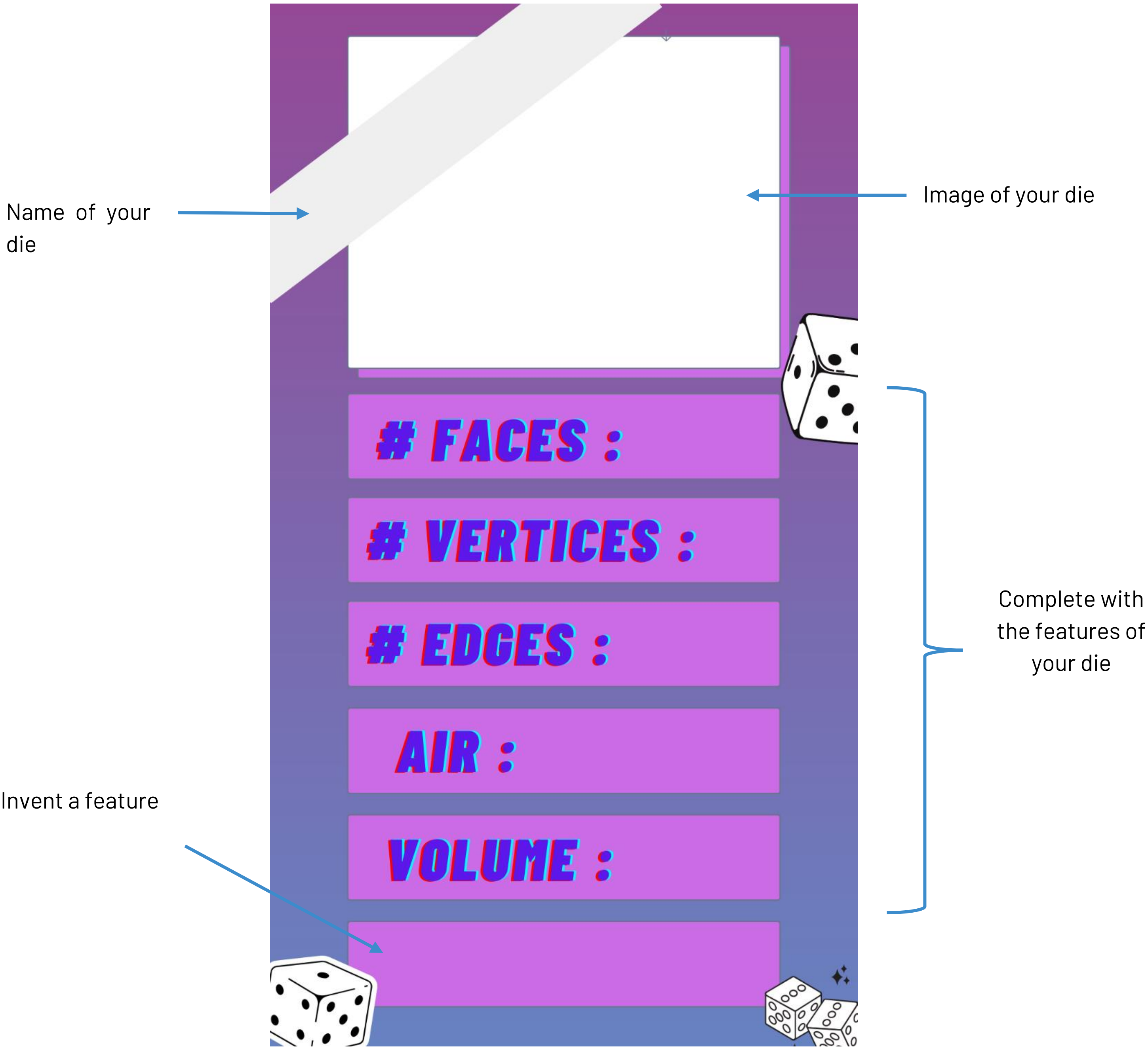
Option 1: Poster

Present your die with a creative poster highlighting its unique features!

▪ Give your die a name and provide an image!	
▪ Specify the number of vertices, edges and faces! Specify the shape of its faces!	[See mini lesson 2]
▪ Construct the net of the solid!	[See mini lesson 3] [See mini lesson 6]
▪ Calculate its total area!	[See mini lesson 4] [See mini lesson 7]
▪ Calculate its volume!	[See mini lesson 5] [See mini lesson 7]
▪ Estimate the probabilities of the different faces of your die!	[See mini lesson 8]

2: Top Trumps

Create a Top Trumps game card. Use the following template in paper or digital form. With the digital version, you're free to change it as you like.



Digital version on Canva

Link

Top Trumps game rules

Top Trumps is a comparative card game. Each pack is themed (cars, planes, famous people, animals, etc) and generally contains between 30 and 40 cards. In this case, the theme will be dice.

Preparation:

- Hand out all the cards face down to the players.
- Each player forms a pile of cards without looking at them.

Sequence of play:

- The first player takes the top card from their pile and chooses a feature/statistic from those shown on their card (for example: # of faces).
- All players then reveal their top card and compare the value of the chosen feature.
- The player with the highest value for this characteristic wins all the cards played during this round and places them under their pile.
- The winner becomes the active player and chooses the characteristic for the next round.
- In the event of a tie between the best cards, the cards concerned are set aside and a new round is played. The winner of this new round also wins the discarded cards.

The aim of the game is to collect all the cards in the deck. A player is eliminated when they run out of cards. The last player standing is declared the winner.

Option 3: Video

Use your 3D-printed die and make a video in which you check a feature of your die in an experimental way. Here are a few ideas (do some research to find out how):

- Determining the volume of the die using the water displacement method (Archimedes' principle)
- Determining the total area of the die by covering it with a material
- Estimating the probabilities of the different faces of your die
- Checking Euler's formula

Phase 5: Creating the game

To come up with ideas for the rules of the game, try answering the following questions:

- Do you want to create a game of chance (like games 1 and 3) or a game with elements of strategy (like games 2, 4 and 5)?
- What will the aim of the game be?
- How many players will there be?
- How many dice will each player have?

This part of the lesson can be done in collaboration with the French or Digital Sciences teacher (see Section *3.4 Interdisciplinary ideas*).

Mini lessons

Mini lesson 1: Tinkercad tutorials

The videos in the following link show the basics of the Tinkercad program.

Tutoriels
Tinkercad

Link

Mini lesson 2: Vocabulary of solids

There are many standard dice shapes.

You can find a selection of the most common models in the list below:

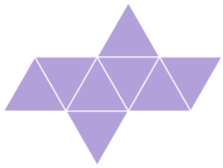
D4 D6 D8 D10 D12 D20

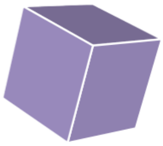
Exercise ML2-1

Here is a list of **solids** and **nets** (or **developments**). Link the solids to their corresponding net.

A

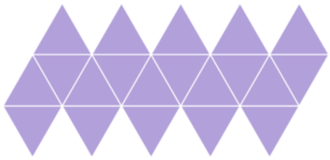
1





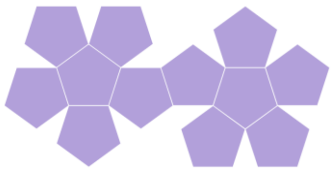
B

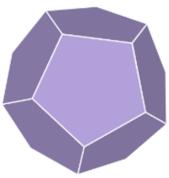
2



C

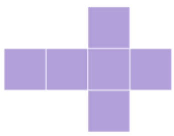
3





D

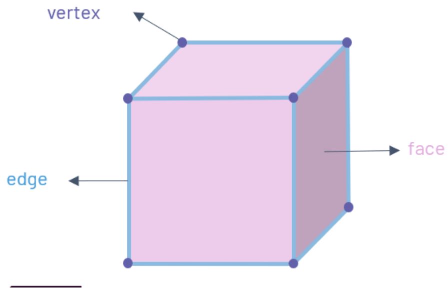
4



E

5

A **polyhedron** is a solid bounded by **plane faces** that are **polygons**. The **vertices** and **edges** of a polyhedron are respectively the vertices and sides of the polygons that bound it.



For some standard dice, the faces are **regular polygons**: geometric shapes whose **sides** all have **the same length** and whose **angles** all have the same **same size**. These solids are called **regular polyhedra** or Platonic solids.

Exercise ML2-2

Complete the table below:

Image of the die	Geometric shape of a face	Number of faces	Number of vertices	Number of edges	Name of the solid	Is this a Platonic solid?
						
						
					Octahedron	
		10			Pentagonal trapezoid	
		12			Dodecahedron	
		20			Icosahedron	

Exercise ML2-3

For each die in the table, call

- F: the number of faces
- V: the number of vertices
- E: the number of edges

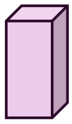
Complete the following table

Image of the die	F	V	E	$F + V - E$
				
				
				
	10			
	12			
	20			

What do you notice?

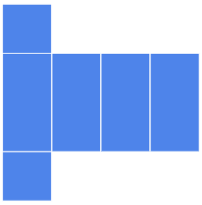
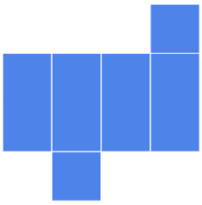
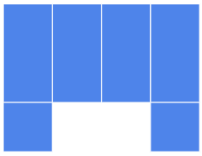
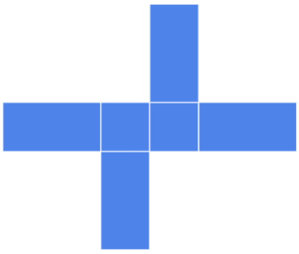
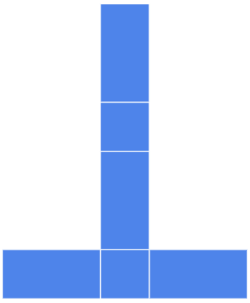
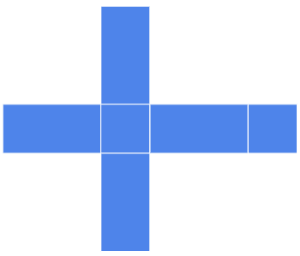
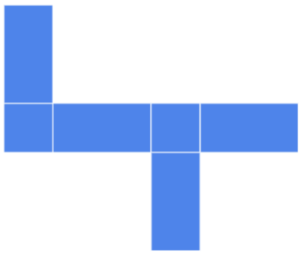
Mini lesson 3: Right prisms

There is another family of solids: **right prisms**. The best-known member of this family is the **cuboid**.



Exercise ML3-1

From among the nets proposed, choose those which, after folding, make it possible to construct a right block.

 Yes ☐ No ☐	 Yes ☐ No ☐	 Yes ☐ No ☐
 Yes ☐ No ☐	 Yes ☐ No ☐	 Yes ☐ No ☐
 Yes ☐ No ☐	 Yes ☐ No ☐	 Yes ☐ No ☐

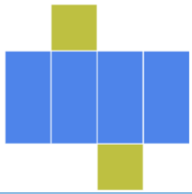
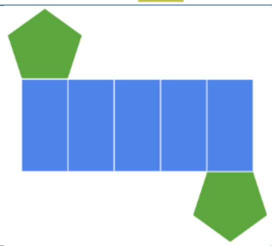
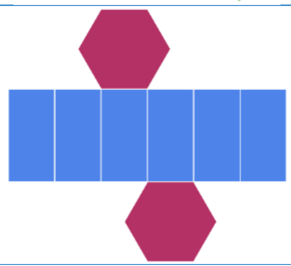
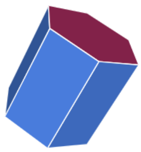
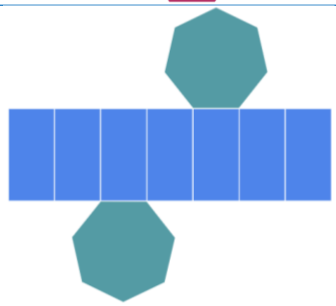
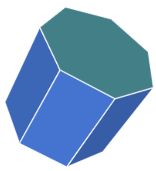
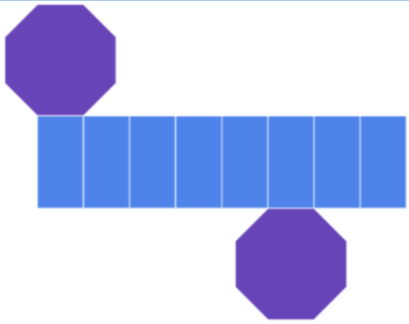
Check your answers using the following application Mathigon :

In general, you can remember that :

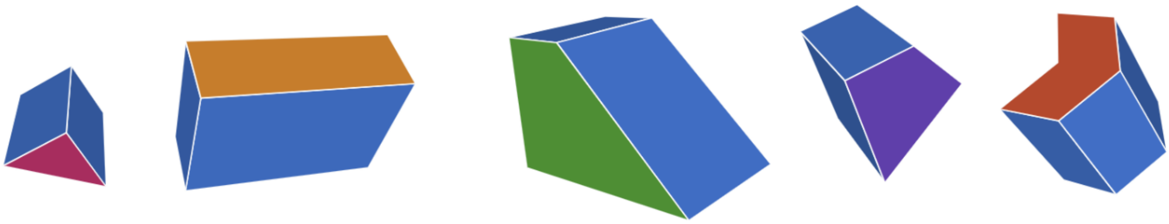
The **sides of** a right prism **are rectangles**. The **two bases of** a right prism **are isometric polygons**.

Exercise ML3-2

Complete the following table:

Base	Net	Right prism
Equilateral triangle		
Square		
		
		
		
		

The bases of a right prism are not necessarily regular polygons. In general, the bases are any polygonal shapes:

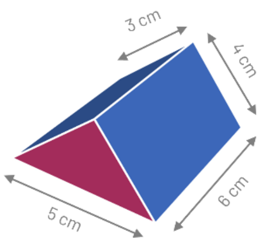
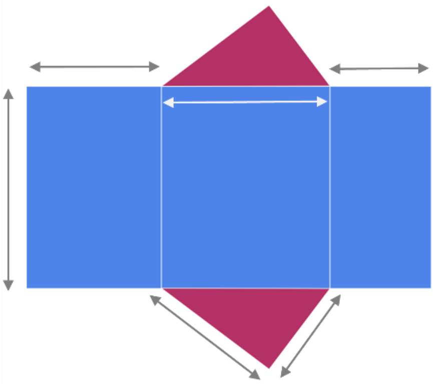
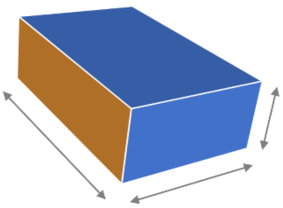
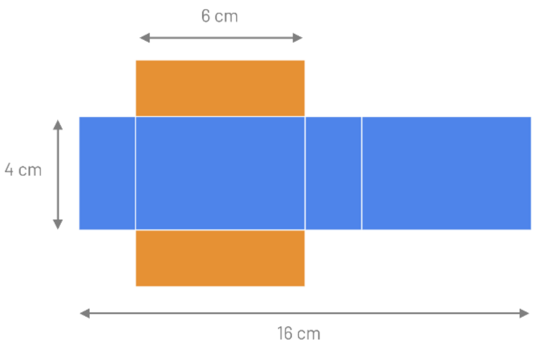


The side faces of these right prisms **are rectangles** that all have **the same height** (in particular the height of the right prism) but not necessarily all the same width.

Examine the net of these right prisms at Mathigon :

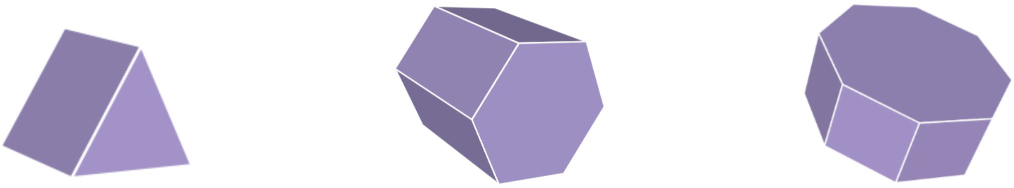
Exercise ML3-3

Use the information available to fill in the missing lengths:

Solid	Net
	
	

Exercise ML3-4

Use the following application Mathigon to construct the proposed solids from their nets:



Mini lesson 4: Area of a right prism

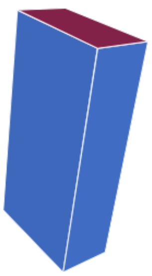
Use the following application Mathigon to calculate **the total area of** the two right prisms proposed.

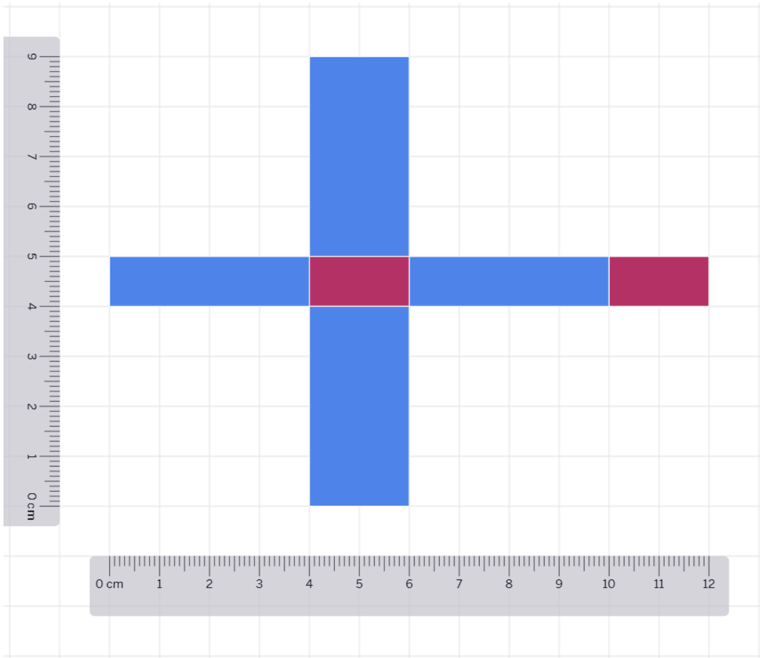
In general, you can remember that :

The **total area** of a **right prism** is equal to the **sum of** the **area** of the **two bases** and the **area of** the **side faces**.

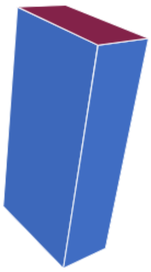
Exercise ML4-1

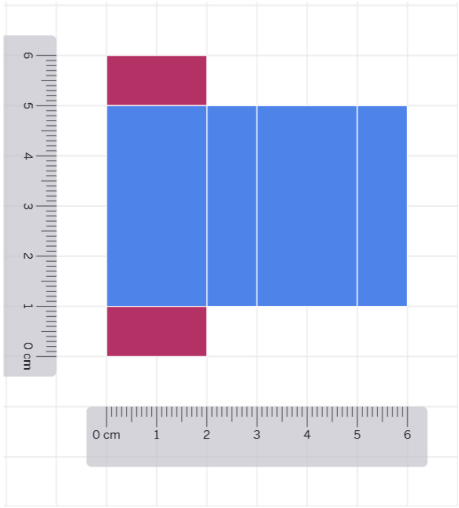
Calculate the total area of the following right prism by examining the proposed net:





To compute the total area of a right prism, it is easiest to use a net that arranges the **side faces** side by side so that they form **a single rectangular surface**.





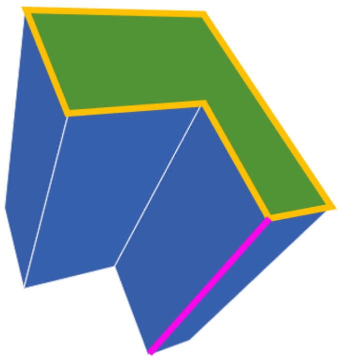
What is the **height of** the right prism?
What is the **perimeter** of one of its **bases**?
What are the **dimensions** of the **rectangle** formed by the **side faces**?
Calculate the area of the side faces:
Calculate the area of the two bases:
Calculate the total area :

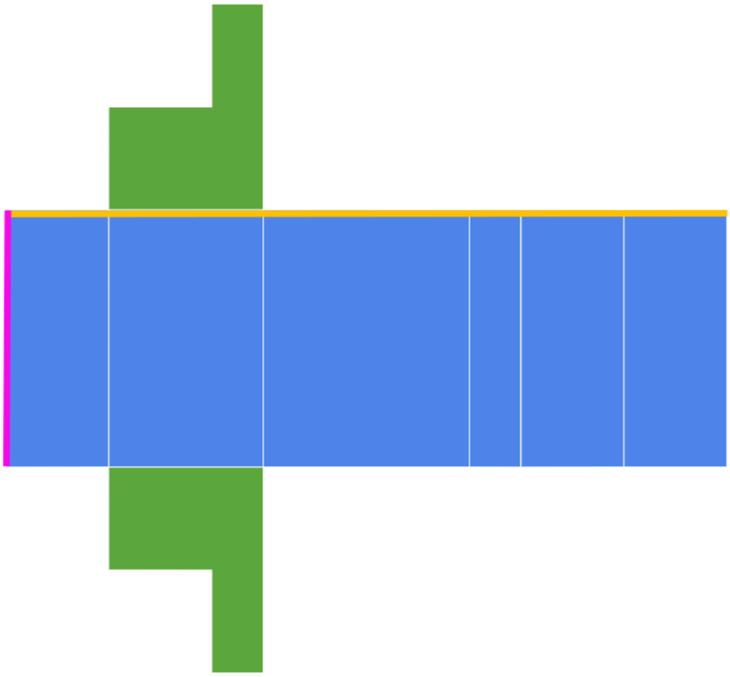
The **area of the sides of** a right prism is equal to the **product of** the **perimeter of the base** and the **height of** the prism.

$$\mathcal{A}_{\text{lateral}} = \mathcal{P}_{\text{base}} \cdot \text{height}$$

The **total area** of a prism is equal to **the sum** of the **area** of the two **bases** and the **lateral area**.

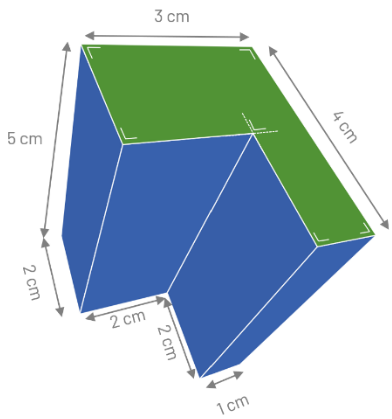
$$\mathcal{A}_{\text{total}} = 2 \cdot \mathcal{A}_{\text{base}} + \mathcal{A}_{\text{lateral}}$$





Example:

Consider the following right prism.



- The perimeter of the base is given by :

$$\begin{aligned}\mathcal{P}_{\text{base}} &= 3 + 4 + 1 + 2 + 2 + 2 \\ &= 14 \text{ cm}\end{aligned}$$

- The lateral area is :

$$\begin{aligned}\mathcal{A}_{\text{lateral}} &= \mathcal{P}_{\text{base}} \cdot h \\ &= 14 \cdot 5 \\ &= 70 \text{ cm}^2\end{aligned}$$

- The area of the base is :

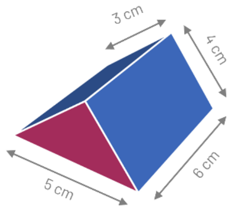
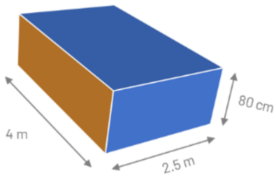
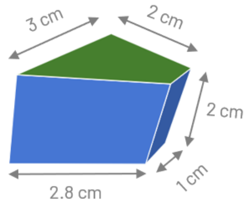
$$\begin{aligned}\mathcal{A}_{\text{base}} &= 2 \cdot 3 + 2 \cdot 1 \\ &= 6 + 2 \\ &= 8 \text{ cm}^2\end{aligned}$$

- Total area is :

$$\begin{aligned}\mathcal{A}_{\text{total}} &= 2 \cdot \mathcal{A}_{\text{base}} + \mathcal{A}_{\text{lateral}} \\ &= 2 \cdot 8 + 70 \\ &= 16 + 70 \\ &= 86 \text{ cm}^2\end{aligned}$$

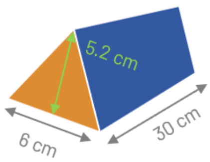
Exercise ML4-2

Calculate the total area of the following right prisms:

Right prism			
Shape of base	Right triangle	Rectangle	Rectangular trapezium
Height			
Perimeter of base			
Base area			
Lateral area			
Total area			

Exercise ML4-3

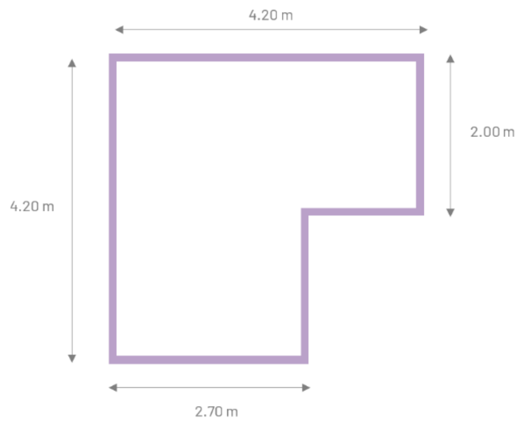
1) A famous type of Swiss chocolate is packaged in a box in the shape of a right prism whose bases are equilateral triangles.



   a) Calculate the total area of the cardboard needed to make this packaging.

   b) How many packages can be made using 1 m^2 of cardboard?

2) Pablo wants to paint the walls and ceiling of his workshop navy blue.
   The walls are 2.5 m high. You can forget about doors and windows when calculating the surface. All the corners of the room are right angles.



   a) Help Pablo calculate the total surface area to be painted.

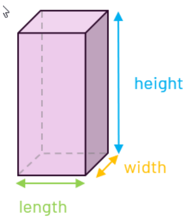
   b) A 2.5 litre pot of paint costs €30 and covers an area of 25 m^2. Help Pablo work out the total cost of the paint.

Mini lesson 5: Volume of a right prism

Use the following application Mathigon to calculate the **volume of** the three **cuboids**.

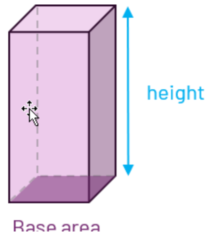
A formula for calculating the **volume** of a **rectangular cuboid** is:

$$\mathcal{V}_{rectangular\ cuboid} = \text{length} \cdot \text{width} \cdot \text{height}$$

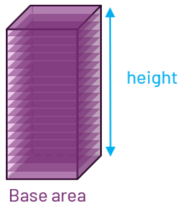


You can also rewrite this formula using **the area of** the rectangular **base**:

$$\mathcal{V}_{\text{pavé droit}} = \underbrace{\text{length} \cdot \text{width}}_{=\mathcal{A}_{\text{base}}} \cdot \text{height}$$
$$= \mathcal{A}_{\text{base}} \cdot \text{height}$$



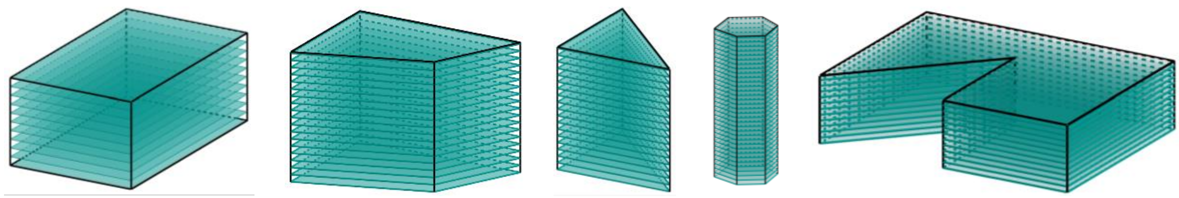
This formula can be explained using the following mental image: imagine the area of the base as an extremely thin sheet of paper of zero thickness. Then, in order to fill the entire right-angled block, you place several of these sheets on top of each other, each with an area identical to that of the base. How many will it take to fill the entire volume of the block? As many as the height of the solid. So, multiply the area of the base by the height and you get the volume of the right block.



In fact this formula can also be used to calculate the volume of any right prism. In general, you can remember that :

The **volume** of a right prism is equal to the **product of the area of the base** and the **height of** the prism.

$$\mathcal{V}_{\text{right prism}} = \mathcal{A}_{\text{base}} \cdot \text{height}$$



Example:

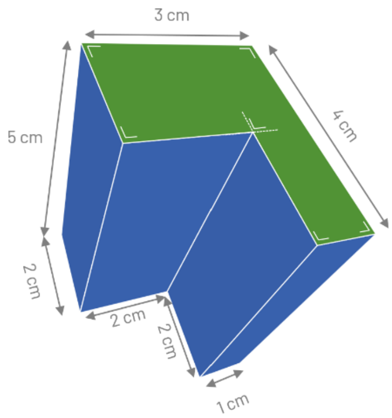
Consider the following right prism.

- The area of the base is :

$$\begin{aligned}\mathcal{A}_{\text{base}} &= 4\cdot 3 - 2\cdot 2\\ &= 12 - 4\\ &= 8\ \text{cm}^2\end{aligned}$$

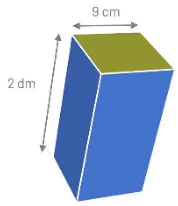
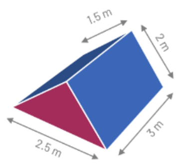
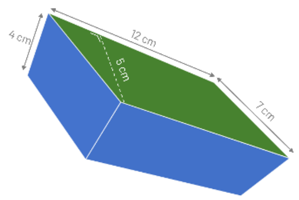
- The volume is :

$$\begin{aligned}\mathcal{V} &= \mathcal{A}_{\text{base}}\cdot \text{height}\\ &= 8\cdot 5\\ &= 40\ \text{cm}^3\end{aligned}$$



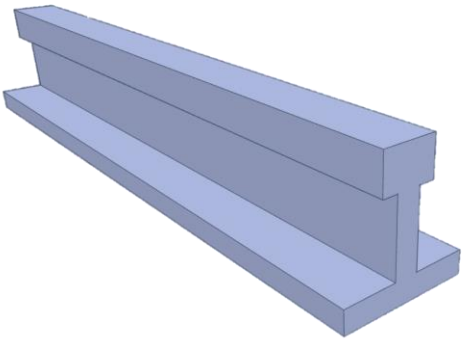
Exercise ML5-1

Calculate the volume of the following right prisms:

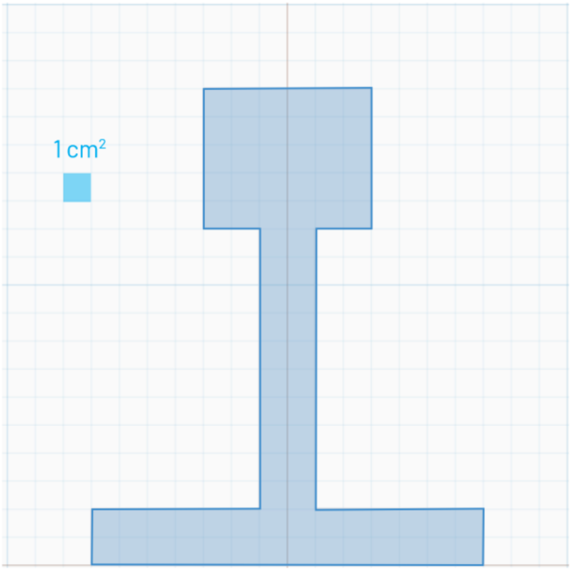
Right prism			
Shape of the base	Square	Right-angled triangle	Parallelogram
Height			
Area of base			
Volume			

Exercise ML5-2

According to the Arcelor-Mittal product catalogue[8] , one metre of type UIC60 rail weighs 60.21 kg. Check this value using the drawings below, knowing that the density of the steel used for railway rails is 7500 kg/m^3.



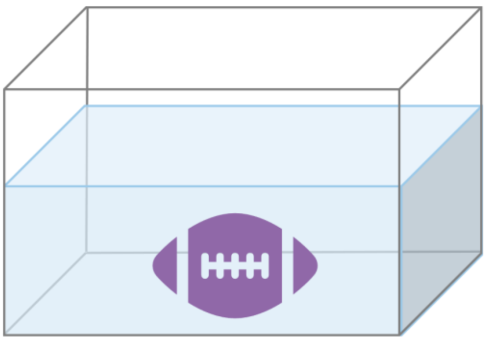
Railway rail (simplified model)



Cross-section

Exercise ML5-3

To determine the volume of an American football, Jerry fills his old fish tank with water to a height of 20 cm. The base of the aquarium is a rectangle measuring 60 cm by 20 cm. He then submerges the football completely in the water. He notices that the water level rises to 23.8 cm from the bottom of the aquarium.



1) Help Jerry calculate the volume of the football.
2) Unfortunately, when Jerry takes the football out of the water, his smartphone falls into the fish tank. He has the latest model which measures 160 mm x 80 mm x 10 mm. Help Jerry to calculate the new water level in the aquarium.

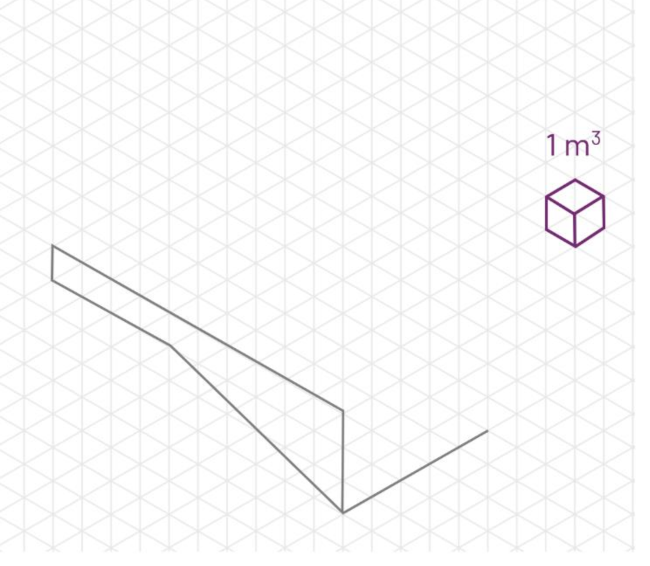
[8] https://rails.arcelormittal.com/wp-content/uploads/2023/10/ArcelorMittal-Transport-Rails-EN.pdf

Exercise ML5-4

Mr PITT has asked an architect to plan an outdoor swimming pool for his new house.

Mr PITT has specific visions for the shape of his new swimming pool. He wants his pool to meet the following requirements:

- The water surface is rectangular, measuring 10 m by 6 m. The vertical walls are perpendicular to the bottom of the pool.
- At the start of the pool, there will be a constant depth of 1 m over the entire width and a length of 4 m.
- After 4 m, the depth increases steadily to reach a maximum depth of 3 m at the other end of the pool.

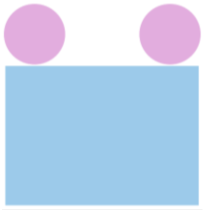
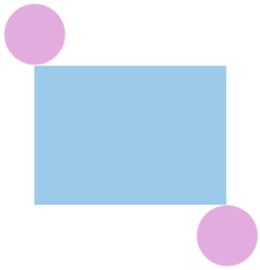


1) The architect has already started to draw a 3D plan of the pool for Mr PITT's swimming pool. Help him to finalise the drawing and enter all the relevant measurements.
2) According to local regulations, it is forbidden to build private swimming pools with a capacity of more than 1000 hl (1 hl = 100 l) of water. Check whether Mr PITT's pool will be granted planning permission.
3) To ensure that the pool is watertight, a plastic film will be placed on the bottom and walls of the pool. Gives a reasonable estimate of the surface area of waterproof film required.
4) Once completed, the pool will be filled using a regular tap with a flow rate of 12 L/min. Calculate the time needed to fill the pool.

Mini lesson 6: Right cylinders

A can of *Pink Horse* is shaped like a **right cylinder**.

Among the proposed nets, mark by those that can correspond to the net of the cylinder. Mark by the nets that cannot be folded to form a cylinder.

Net A	Net B	Net C
		
Net D	Net E	Net F
		

Complete the text below:

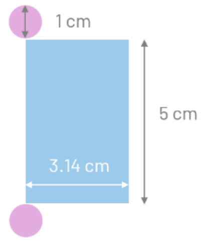
- The nets __________________ correspond to the net of a cylinder.
- The bases of a cylinder are _______________________ and the side face is _______________________ .

In general, you can remember that :

The **side of** a right cylinder is a **rectangle**. The two **bases of** a cylinder **are discs of the same radius**.

Exercise ML6-1

Here are the nets of three right cylinders. Match the nets to the cylinders and fill in the missing measurements:

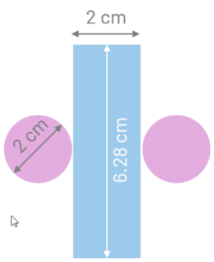


Base radius :

$r =$ ___________

Height :

$h =$ ___________

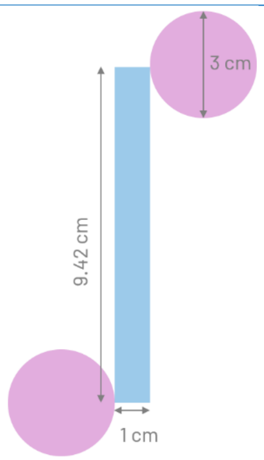


Base radius :

$r =$ ___________

Height :

$h =$ ___________



Base radius :

$r =$ ___________

Height :

$h =$ ___________

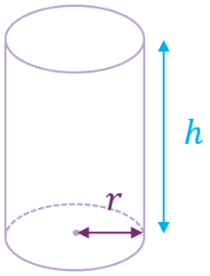
What does the 3rd measurement shown, which does not represent the diameter of the base or the height of the cylinder, represent?

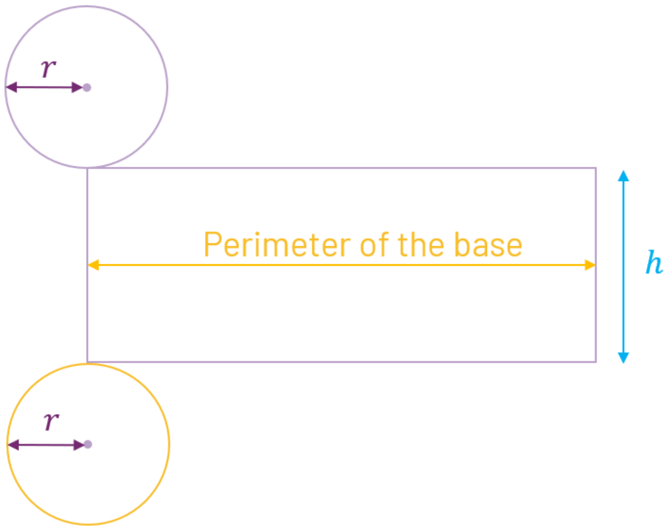
- The radius of the base is equal to _________________ of the diameter of the base.
- The height of the rectangular side is ________________ of the cylinder.
- The length of the rectangular side is _________________ of the base.

In general, you can assume that :

For a right cylinder with radius r and height h :

- The **side face** is a rectangle of height h and length equal to the **perimeter of the base**.
- The two **bases** are discs of radius r.





Mini lesson 7: Area and volume of a cylinder

The principles we have seen for calculating the area and volume of a right prism also apply to a cylinder.

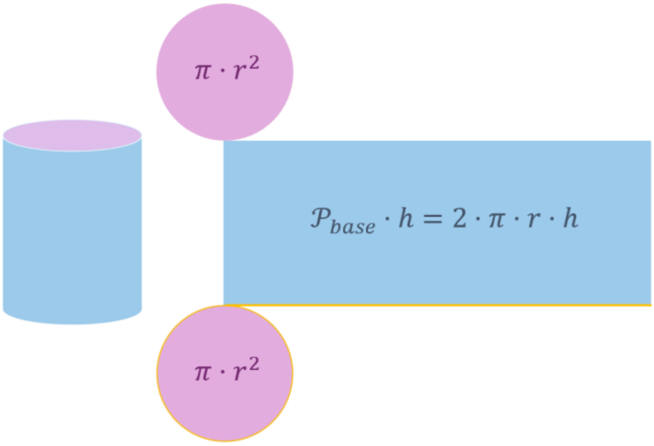
In general, you can remember that :

- The **area** of the **side of** a cylinder is equal to the **product of** the **perimeter** of the **base** and the **height of** the cylinder.

$$\mathcal{A}_{\text{lateral}} = \mathcal{P}_{\text{base}} \cdot \text{height}$$

- The **total area** of a cylinder is equal to the **sum** of the **area** of the two **bases** and ***the* lateral area**.

$$\mathcal{A}_{\text{total}} = 2 \cdot \mathcal{A}_{\text{base}} + \mathcal{A}_{\text{lateral}}$$



Lateral area of a right cylinder :

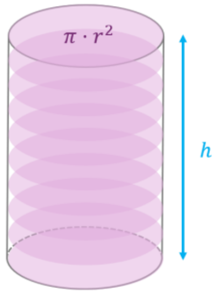
$$\begin{aligned}\mathcal{A}_{\text{lateral}} &= \mathcal{P}_{\text{base}} \cdot \text{height} \\ &= 2 \cdot \pi \cdot r \cdot h \\ &= 2\pi r h\end{aligned}$$

Total area of a right cylinder :

$$\begin{aligned}\mathcal{A}_{\text{total}} &= 2 \cdot \mathcal{A}_{\text{base}} + \mathcal{A}_{\text{lateral}} \\ &= 2 \cdot \pi \cdot r^2 + 2 \cdot \pi \cdot r \cdot h \\ &= 2\pi r^2 + 2\pi r h\end{aligned}$$

- The **volume** of a **cylinder** is equal to the **product of the area of the base** and the **height of** the cylinder.

$$\mathcal{V}_{\text{cylinder}} = \mathcal{A}_{\text{base}} \cdot \text{height}$$

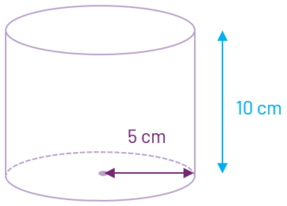
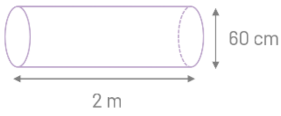
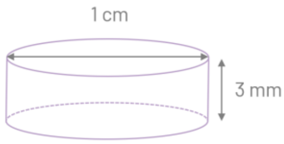


Volume of a right cylinder:

$$\begin{aligned}\mathcal{V}_{\text{cylinder}} &= \mathcal{A}_{\text{base}} \cdot \text{height} \\ &= \pi \cdot r^2 \cdot h \\ &= \pi r^2 h\end{aligned}$$

Exercise ML7-1

Calculate the volume and total area of the following right cylinders:

Cylinder			
Area of base			
Perimeter of base			
Lateral area			
Total area			
Volume			

Exercise ML7-2

We want to compare the volume and total area of three objects:

Object 1	Object 2	Object 3
A 50 cent coin	One CR2032 battery	A single strand of spaghetti
		
ø 24 mm ; h 2.4 mm	ø 20 mm ; h 3.2 mm	ø 2 mm ; h 25 cm

Without doing any **calculations**, try to rank the volume and total area of these objects in ascending order:

$$\mathcal{V}_{.....} \quad < \quad \mathcal{V}_{.....} \quad < \quad \mathcal{V}_{.....}$$

$$\mathcal{A}_{.....} \quad < \quad \mathcal{A}_{.....} \quad < \quad \mathcal{A}_{.....}$$

Check your answer by assuming that these objects are right cylinders and **doing some calculations**. Adjust your classification if necessary:

$$\mathcal{V}_{.....} \quad < \quad \mathcal{V}_{.....} \quad < \quad \mathcal{V}_{.....}$$

$$\mathcal{A}_{.....} \quad < \quad \mathcal{A}_{.....} \quad < \quad \mathcal{A}_{.....}$$

Exercise ML7-3

1) Calculate the volume of PVC needed to produce a DN160 pipe with a length of 2 m, an external diameter of 160 mm and a wall thickness of 4 mm.

2) A wooden plate measuring 100 cm x 100 cm x 18 mm has 190 holes with a diameter of 8 mm. Calculate the weight of the perforated plate, if you know that the wood used has a density of 700 kg/m^3.

3) Calculate the diameter of a can of peeled tomatoes, knowing that the can will contain 500 ml and that the height of the can is 11 cm.

Mini lesson 8: Dice and probabilities

Go to mathigon.com and simulate 500 throws of a D4 die. Count the number of times the die shows a 1, 2, 3 or 4. Then divide the number of times each result appears by 500 to calculate the frequency.

500 throws	1	2	3	4	Total
					
Frequency					

You can see that the frequencies are all approximately equal to _________.

For a D4 die, you can conclude that the theoretical frequency of getting a 1, 2, 3 or 4 is __________________ .

Repeat the experiment with a D6 die using the following link:

500 throws	1	2	3	4	5	6	Total
							
Frequency							

You can see that the frequencies are all approximately equal to _________.

For a D6 die, you can conclude that the theoretical frequency of getting a 1, 2, 3, 4, 5 or 6 is __________________ .

Complete the following sentences:

The theoretical frequency of getting a 7 with a D8 is __________________ .

The theoretical frequency of obtaining a 10 with a D10 is __________________ .

The theoretical frequency of obtaining a 3 with a D20 is __________________ .

3.4 Interdisciplinary ideas

Digital Sciences

The fifth stage of the "die-sign" project invites students to design their own game. This creative activity can be carried out in collaboration with the Digital Sciences teacher or integrated directly into this course.

This approach is particularly relevant given that the 4th thematic focus of the Digital Sciences syllabus[9] deals precisely with these themes. There are even teaching sheets to support students in the process of creating a game.

The link between this module and the 4th thematic focus of the Digital Sciences course represents an excellent opportunity to decompartmentalise learning, enabling pupils to apply the skills they have acquired in one subject in another educational context.

French class

After designing their game, the pupils have to formalise the rules in writing. This provides an excellent opportunity for collaboration with the French teacher. This activity fits in perfectly with the French curriculum of 6^{e}, which identifies writing a variety of texts within the constraints of the exercise as a subject-specific skill. The rules of the game present a particular style of writing, characterised by precision, clarity and logical structure.

Therefore, writing rules is a relevant pedagogical exercise for French lessons. It enables pupils to familiarise themselves with a standardised textual genre and its codes, while giving their written work a concrete purpose, as it will be used directly in their game creation.

[9] file:///C:/Users/ann.kiefer/Downloads/PROG_6C_DIGSC.pdf

3.5 More on this topic

01 Alea jacta est

The dice embodies chance, luck and the irrevocability of decisions taken. Its precise origin remains enigmatic, with several ancient civilisations - Indian, Egyptian, Roman and Greek - taking credit for its invention. The ancestors of the dice, made of fruit stones, teeth or stones, appeared around 6000 BC and were generally only two-sided (MacDonald, 2018).

Notable precursors include "astragals", ossicles taken from the ankles of animals such as sheep and goats. These six-sided objects were used in games of skill and chance in the ancient Greek and Roman worlds. The Omilla game, which was particularly popular, consisted of throwing these astragals into a circle to expel the opponents' figurines - a practice that continues today in the form of the game of marbles played by children in playgrounds (Hennewig, 2022).

Source: https://www.deutsche-digitale-bibliothek.de/content/blog/die-wuerfel-sind-gefallen-geschichte-und-geschichten-eines-spielgeraets-und-symbols/

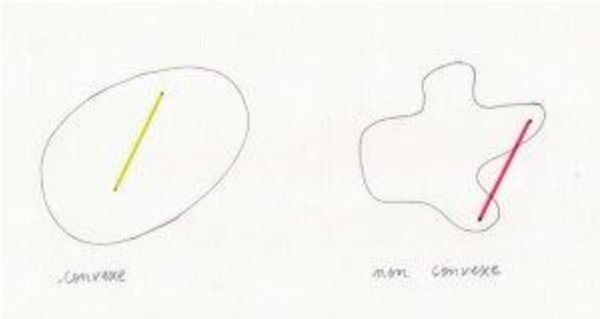
When used as dice, astragals had a particularity: despite having six sides like modern dice, their probabilities were not equivalent. Two sides, because of their configuration, could not be used as a bearing surface, while the other four, all different, offered varying probabilities of appearing during a throw.

Until the Renaissance, the equiprobability of results was not considered important. The concepts of chance and probability did not yet exist, and the choice of which face appeared after a throw was attributed to a divine power.

This paradigm changed with the advent of thinkers such as Galileo and Blaise Pascal who, partly through the study of games and dice, developed the theories of chance and probability. These notions quickly became embedded in the collective consciousness, transforming our perception of dice: today, we simply associate them with luck or misfortune, and naturally use regular solids to guarantee a fair distribution of results (Wissenschaft, 2018).

02 Plato's five solids

The common six-sided die we regularly play with is, mathematically, a cube. A cube is a regular, convex polyhedron, i.e. a three-dimensional geometric shape (a geometric solid) with plane polygonal faces that meet along line segments called edges. A polyhedron is said to be regular if it is made up of faces that are all identical and regular and if all its vertices are identical. Convexity is a mathematical term that, among other things, excludes the presence of holes. A part C of space (or of the plane) is convex if it satisfies the following property: for any pair of points p and q located on C, the segment joining p to q is entirely contained within C. For example, a ball is convex, but a ring is not (Cantat, 2022).



Source: https://images-archive.math.cnrs.fr/La-Gomboc.html?lang=fr

In mathematics, a regular convex polyhedron is called a Platonic solid. In addition to the cube, there are exactly 4 other Platonic solids: the tetrahedron, the octahedron, the dodecahedron and the icosahedron. These are also called Plato's five solids.

In the following video made by the youtube channel Numberphile, mathematicians Katie Steckles and James Grime explain why there are only exactly five Platonic solids:

Numberphile

https://www.youtube.com/watch?v=gVzu1_12FUc

03 The Gömböc

The dice have 6 stable positions on which they can be placed. Stability means that the object remains in the position in which it was placed, even if it is tapped a little. For example, if a bottle is placed vertically, on its bottom, and I tilt it very slightly and release it, then it will return to its initial vertical position by itself and stay there (Cantat, 2022).

Some objects have only one stable balance position, the most emblematic being the tumbler. This egg-shaped toy, weighted at the bottom, is known in its commercial version as the roly-poly toy, which is now somewhat out of fashion.

If you place a tumbler on its head in a perfectly vertical position, it is theoretically in equilibrium: without any external disturbance, it could maintain this position indefinitely. However, this position is unstable - in practice, the slightest disturbance will cause it to tip over and return to its stable position with its feet on the ground. The tumbler therefore has exactly two equilibrium positions: one stable, the other unstable.

This characteristic behaviour is achieved thanks to a lead ball inserted into the lower part of the toy, positioning the centre of gravity at the bottom of the body. The tumbling toy's body is therefore not homogeneous: its internal density varies from one area to another to favour a single stable equilibrium position.

In 1995, the Russian mathematician Vladimir Igorevich Arnold asked the following question.

Arnold's question. *Is there a homogeneous convex body with only two equilibrium positions, one stable, the other unstable?*

The question asks: can we create a solid similar to the roly-poly toy but made of a perfectly homogeneous material? In 2006, two Hungarian researchers succeeded in doing what seemed impossible, providing a positive answer to Arnold's question. Gábor Domokos and Péter Várkonyi, mathematicians from Budapest, designed an entirely homogeneous rocking chair that they named "gömböc". In the following video, Domokos recounts the complex path that led to this remarkable invention.

GÖMBÖC

https://www.youtube.com/watch?v=tD-HgrUChwY

Chandler Davis, editor of The Mathematical Intelligencer, had this to say about the discovery of the gömböc:

A shape whose impossibility might have been an elegant theorem, but whose existence may be much more elegant.

The Gömböc is a solid in dimension 3. Mathematicians have naturally wondered whether such a body also exists in dimension 2. The answer is no.

Theorem. *There is no such thing as a planar gömböc. A convex part of the plane cannot have exactly two equilibrium positions, one stable and one unstable: it always has more!*

For a proof of the theorem, we refer the reader to the article (Cantat, 2022).

04 Polyhedra in research today

Polyhedra belong to a branch of mathematics known as *discrete geometry*, a mathematical field that studies "discrete geometric objects" - i.e. those that can be characterised by a finite number of parameters.

Although this branch is relatively underdeveloped in classical mathematics departments around the world, it occupies a predominant place in computer science departments. This difference in interest can be explained by the twofold attraction it holds for computer scientists: these objects are fascinating in themselves but also offer many practical applications in the computer field.

A common misconception is that discrete geometry is a field without unresolved questions. This perception stems from two main factors: on the one hand, certain fundamental objects such as polyhedra, and particularly the Platonic solids, have been the subject of study since the time of ancient Greek mathematics, suggesting a fully explored field; on the other hand, contemporary problems in discrete geometry often seem excessively technical and inaccessible to non-specialists.

However, contrary to this widespread impression, there are still some open problems in discrete geometry that are remarkably simple to formulate, understandable even without advanced mathematical training, but whose solution continues to challenge researchers. Here are two such open problems.

Unfolding a polyhedron

In this module, we saw that solids have nets, which we cut out, fold along the edges and then glue back together to obtain the original polyhedron. This leads to a very natural question:

Given a polyhedron, can it always be obtained in this way?

In slightly more precise terms, is it always possible to cut a (convex) polyhedron along certain edges, to obtain a connected domain that can be unfolded on the plane (without self-intersection)? Well, we still don't know (Schlenker, 2009).

Subdividing the cube

Is it possible to subdivide a cube into tetrahedra whose angles are all acute, i.e. strictly less than 90 degrees? In dimension 2, the answer is yes: we can divide a square into acute triangles. In dimension three, well, we don't know (Schlenker, 2009)!

1. Cantat, S. 2022. La Gömböc. *Images des Mathématiques.* https://images-archive.math.cnrs.fr/La-Gomboc.html?lang=fr

2. MacDonal, J. 2018. The Ancient Origins of Dice. *JSTOR* Daily. https://daily.jstor.org/the-ancient-origins-of-dice/

3. Schlenker, J-M. 2009. Quelques problèmes ouverts de géométrie élémentaire. *Images des Mathématiques.* https://images.math.cnrs.fr/billets/10766/

4. Wissenschaft. 2018. Wie Würfel-Würfe wirklich zufällig wurden. https://www.wissenschaft.de/geschichte-archaeologie/wie-wuerfel-wuerfe-wirklich-zufaellig-wurden/
5. Hennewig, L. 2022. "Die Würfel sind gefallen." Geschichte und Geschichten eines Spielgeräts und Symbols. *Deutsche Digitale Bibliothek.* https://www.deutsche-digitale-bibliothek.de/content/blog/die-wuerfel-sind-gefallen-geschichte-und-geschichten-eines-spielgeraets-und-symbols/

3.6 A word from the scientists

In this section, we highlight two mathematicians who specialise in applied mathematics, a field that adapts mathematical concepts to various disciplines and professional sectors. First we look at Rima Alaifari, an Austrian-born mathematician, followed by Ingrid Daubechies, the eminent Belgian mathematician who supervised Rima's thesis.

01 Rima Alaifari

Rima Alaifari is Assistant Professor in Applied Mathematics at ETH Zurich. Her research interests lie mainly in the areas of applied analytics, inverse problems and scientific machine learning. Her research interests include stability analysis and regularisation of inverse problems, applied harmonic analysis, phase recovery and stability aspects of machine learning, in particular operator learning. She is an associate member of the ETH AI Center.

Her academic career began with studies in applied and industrial mathematics at Johannes Kepler University in Linz, where she obtained her bachelor's and master's degrees between 2005 and 2010. She then continued with a PhD in mathematics at the Vrije Universiteit Brussel from 2010 to 2014, under the supervision of Professors Ingrid Daubechies (see below) and Michel Defrise.

After her PhD, her post-doctoral career continued at ETH Zurich. Since October 2016, she has held the position of Assistant Professor in Applied Mathematics at ETH Zurich, where she continues to develop her research at the intersection of applied mathematics and artificial intelligence. From September 2025, she will be Professor at the University of Aachen in Germany.

Rima Alaifari

https://www.youtube.com/watch?v=l0VE0-Tv78s

02 Ingrid Daubechies

Ingrid Daubechies is a Belgian mathematician with American citizenship, famous for her groundbreaking work on wavelets and image compression.

After obtaining her doctorate in physics from the Vrije Universiteit Brussel in 1980, she pursued a post-doctoral career in the United States before returning to teach theoretical physics at her home university. Her initial research focused on quantum physics operators. In 1987, she settled permanently in the United States, first working at Bell Laboratories and then, in 1994, becoming the first woman professor of mathematics at Princeton University.

Renowned for her innovative mathematical methods improving image compression technologies, Daubechies is a member of prestigious institutions such as the National Academy of Engineering and the US National Academy of Sciences, as well as the American Academy of Arts and Sciences. Her scientific excellence earned her a MacArthur Fellowship in 1992. Between 2011 and 2013, she was a member of the jury for the Infosys Prize for Mathematical Sciences, before joining the Academia Europaea in 2015.

Strongly committed to promoting diversity, Daubechies sits on the steering committee of Enhancing Diversity in Graduate Education, a programme supporting women pursuing postgraduate studies in mathematics. Between 2011 and 2014, she made history by becoming the first female president of the International Mathematical Union.

The following video was made in 2023 when she was awarded the prestigious Wolf Prize.

Ingrid Daubechies
2023 Wolf Prize

https://www.youtube.com/watch?v=Z36zl2iPXgc

The second video is longer. It is a video of an interview at CIRM (Centre International de Rencontres Mathématiques) in Marseille.

Ingrid Daubechies
CIRM

https://www.youtube.com/watch?v=lhR7K6xp2Cg&t=283s

ISSN 2738-9383

PIT

TEACHING AND TRAINING • PROGRAMME FOR INNOVATIVE •