Mini lesson 2: Vocabulary of solids

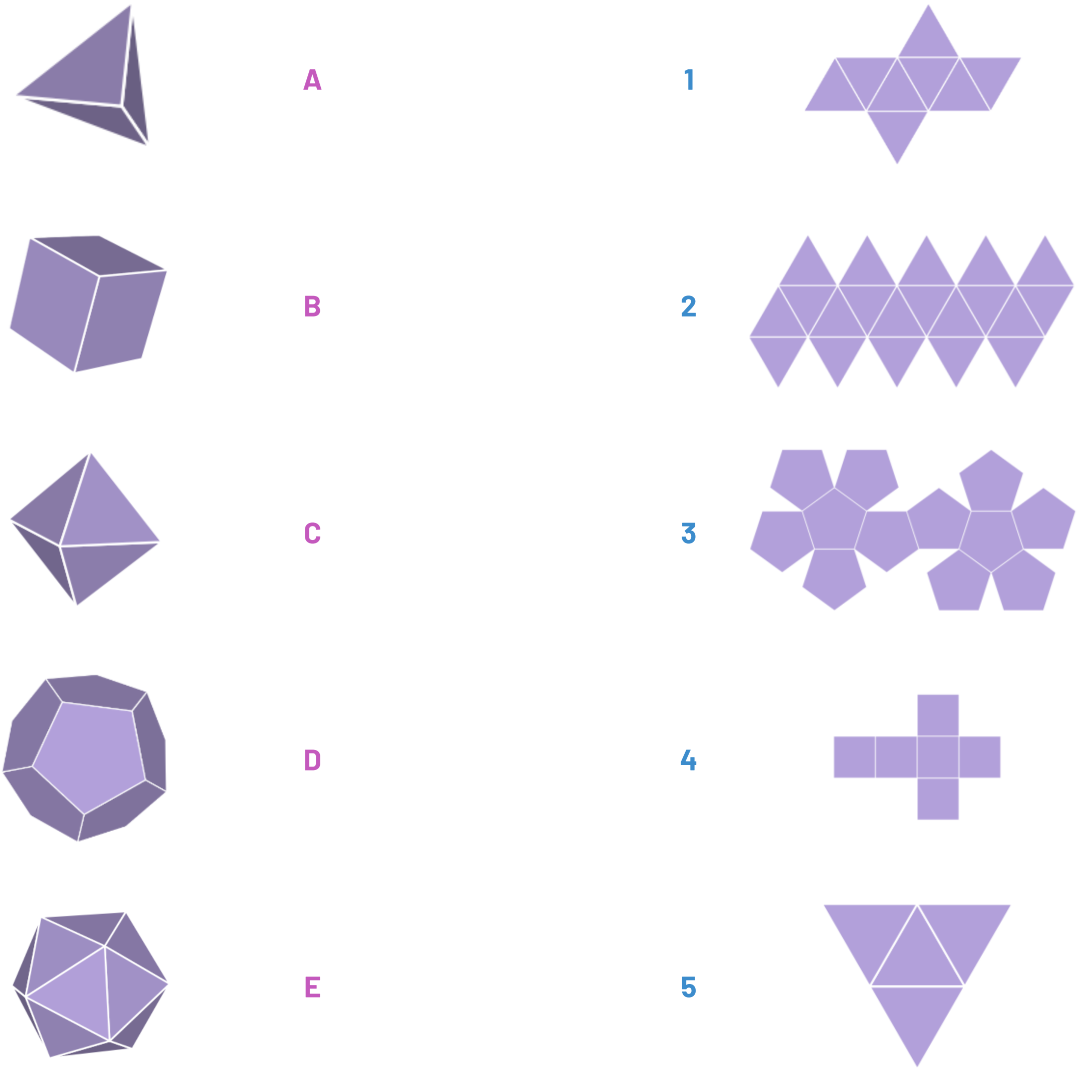
There are many standard dice shapes.

You can find a selection of the most common models in the list below:

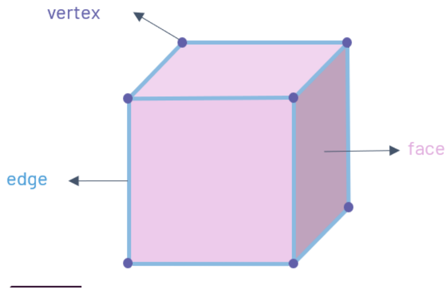
D4 D6 D8 D10 D12 D20

Exercise ML2-1

Here is a list of **solids** and **nets** (or **developments**). Link the solids to their corresponding net.



A **polyhedron** is a solid bounded by **plane faces** that are **polygons**. The **vertices** and **edges** of a polyhedron are respectively the vertices and sides of the polygons that bound it.



For some standard dice, the faces are **regular polygons**: geometric shapes whose **sides** all have **the same length** and whose **angles** all have the same **same size**. These solids are called **regular polyhedra** or Platonic solids.

Exercise ML2-2

Complete the table below:

Image of the die	Geometric shape of a face	Number of faces	Number of vertices	Number of edges	Name of the solid	Is this a Platonic solid?
						
						
					Octahedron	
		10			Pentagonal trapezoid	
		12			Dodecahedron	
		20			Icosahedron	

Exercise ML2-3

For each die in the table, call

- F: the number of faces
- V: the number of vertices
- E: the number of edges

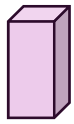
Complete the following table

Image of the die	F	V	E	$F+V-E$
				
				
				
	10			
	12			
	20			

What do you notice?

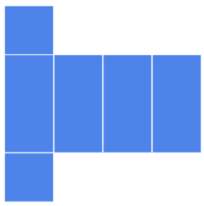
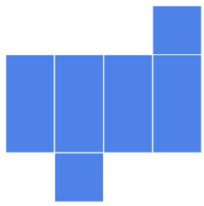
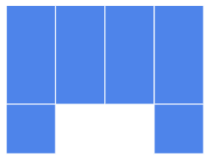
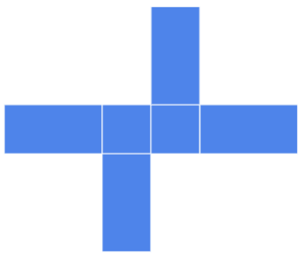
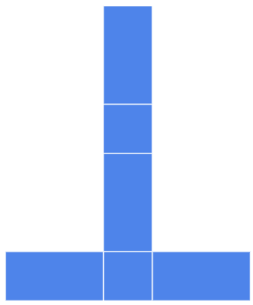
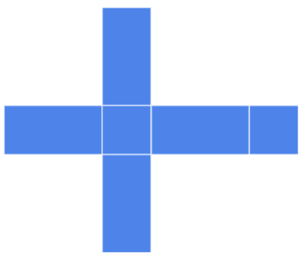
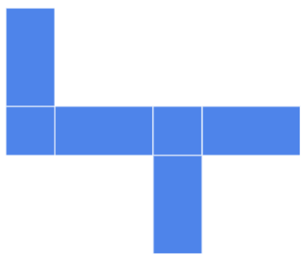
Mini lesson 3: Right prisms

There is another family of solids: **right prisms**. The best-known member of this family is the **cuboid**.



Exercise ML3-1

From among the nets proposed, choose those which, after folding, make it possible to construct a right block.

 Yes ☐ No ☐	 Yes ☐ No ☐	 Yes ☐ No ☐
 Yes ☐ No ☐	 Yes ☐ No ☐	 Yes ☐ No ☐
 Yes ☐ No ☐	 Yes ☐ No ☐	 Yes ☐ No ☐

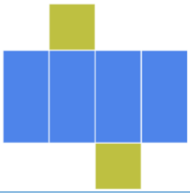
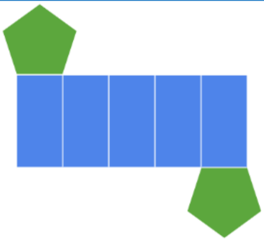
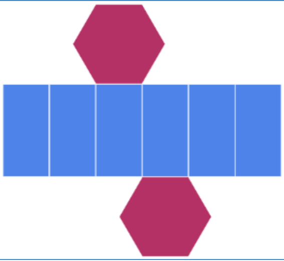
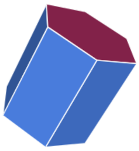
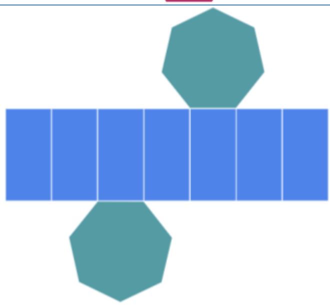
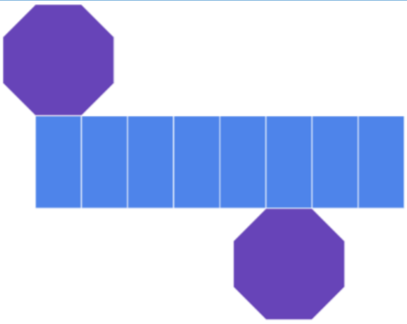
Check your answers using the following application Mathigon :

In general, you can remember that :

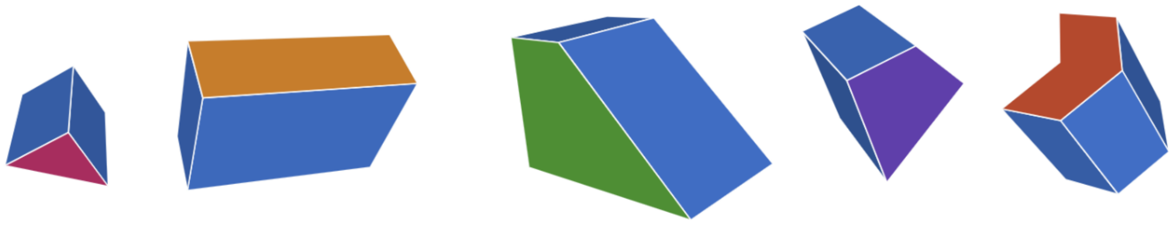
The **sides of** a right prism **are rectangles**. The **two bases of** a right prism **are isometric polygons**.

Exercise ML3-2

Complete the following table:

Base	Net	Right prism
Equilateral triangle		
Square		
		
		
		
		

The bases of a right prism are not necessarily regular polygons. In general, the bases are any polygonal shapes:

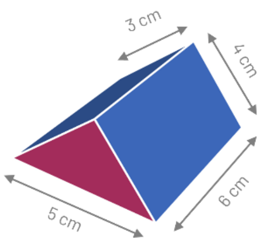
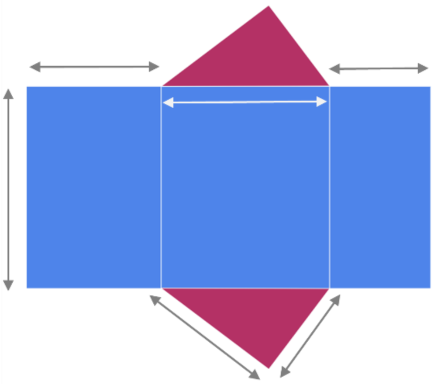
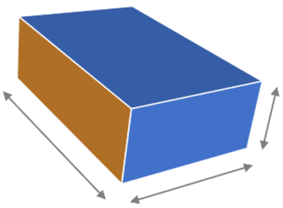
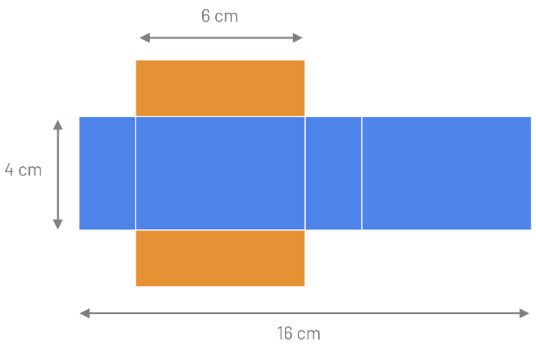


The side faces of these right prisms **are rectangles** that all have **the same height** (in particular the height of the right prism) but not necessarily all the same width.

Examine the net of these right prisms at Mathigon :

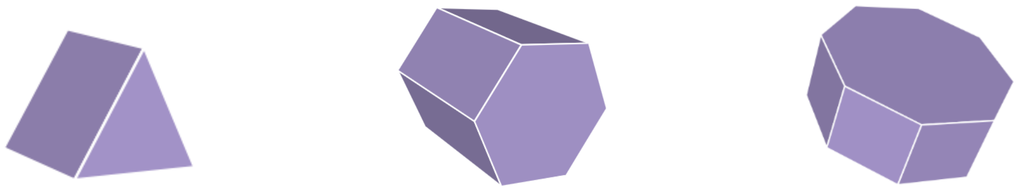
Exercise ML3-3

Use the information available to fill in the missing lengths:

Solid	Net
	
	

Exercise ML3-4

Use the following application Mathigon to construct the proposed solids from their nets:



Mini lesson 4: Area of a right prism

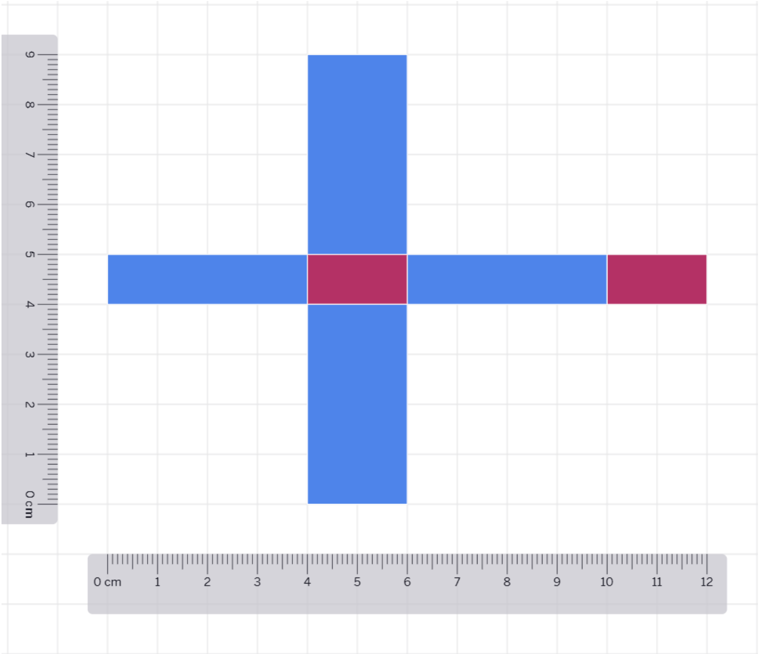
Use the following application Mathigon to calculate **the total area of** the two right prisms proposed.

In general, you can remember that :

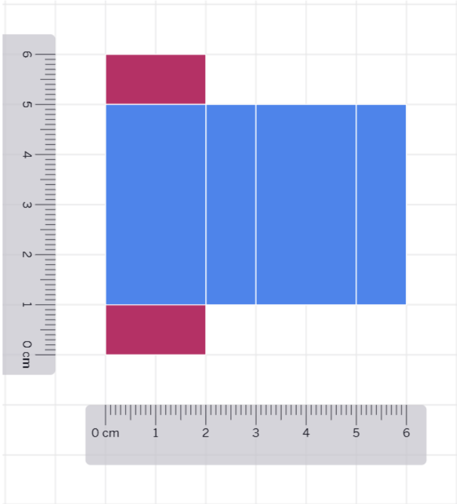
The **total area** of a **right prism** is equal to the **sum of** the **area** of the **two bases** and the **area of** the **side faces**.

Exercise ML4-1

Calculate the total area of the following right prism by examining the proposed net:



To compute the total area of a right prism, it is easiest to use a net that arranges the **side faces** side by side so that they form **a single rectangular surface**.



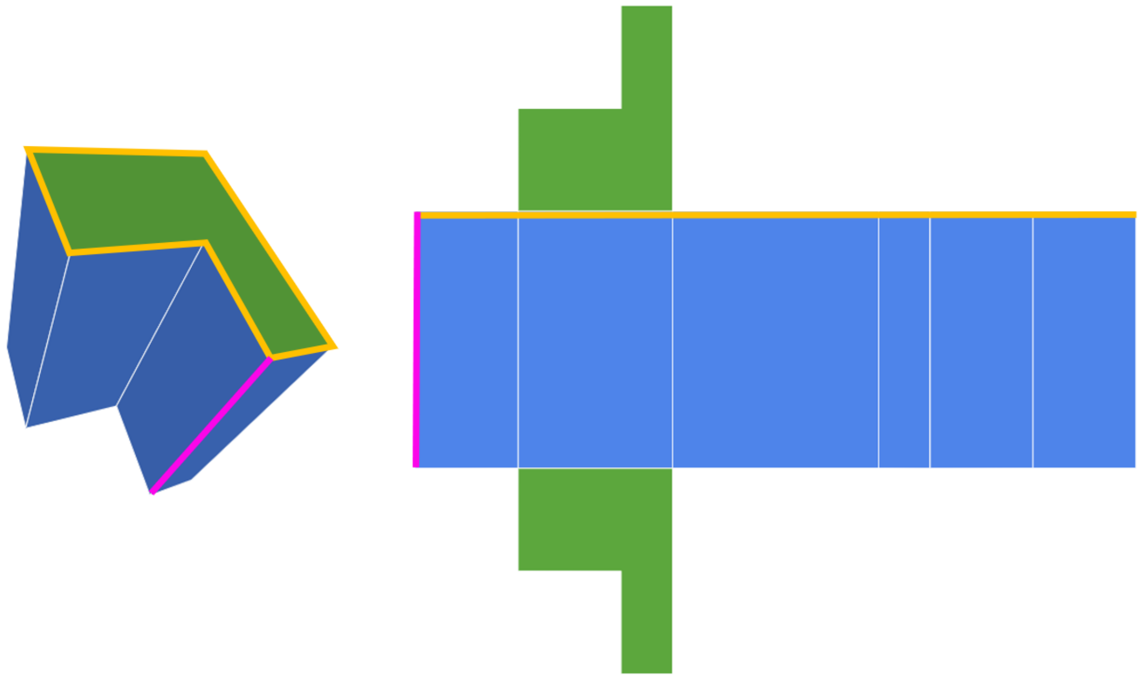
What is the **height of** the right prism?
What is the **perimeter** of one of its **bases**?
What are the **dimensions** of the **rectangle** formed by the **side faces**?
Calculate the area of the side faces:
Calculate the area of the two bases:
Calculate the total area :

The **area of the sides of** a right prism is equal to the **product of** the **perimeter of the base** and the **height of** the prism.

$$\mathcal{A}_{\text{lateral}} = \mathcal{P}_{\text{base}} \cdot \text{height}$$

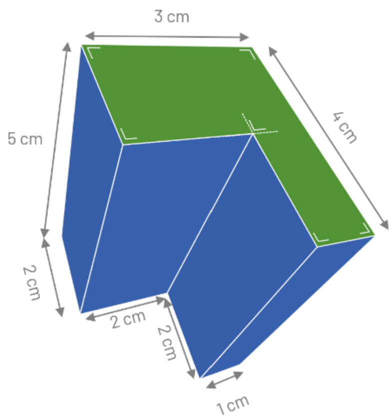
The **total area** of a prism is equal to **the sum** of the **area** of the two **bases** and the **lateral area**.

$$\mathcal{A}_{\text{total}} = 2 \cdot \mathcal{A}_{\text{base}} + \mathcal{A}_{\text{lateral}}$$



Example:

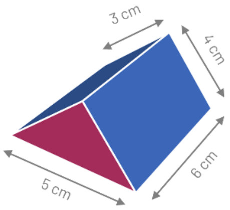
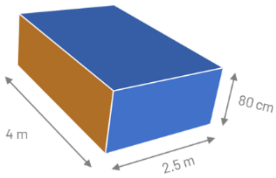
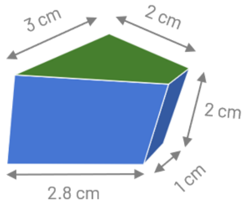
Consider the following right prism.



- The perimeter of the base is given by :
$$\begin{aligned}\mathcal{P}_{\text{base}} &= 3 + 4 + 1 + 2 + 2 + 2 \\ &= 14 \text{ cm}\end{aligned}$$
- The lateral area is :
$$\begin{aligned}\mathcal{A}_{\text{lateral}} &= \mathcal{P}_{\text{base}} \cdot h \\ &= 14 \cdot 5 \\ &= 70 \text{ cm}^2\end{aligned}$$
- The area of the base is :
$$\begin{aligned}\mathcal{A}_{\text{base}} &= 2 \cdot 3 + 2 \cdot 1 \\ &= 6 + 2 \\ &= 8 \text{ cm}^2\end{aligned}$$
- Total area is :
$$\begin{aligned}\mathcal{A}_{\text{total}} &= 2 \cdot \mathcal{A}_{\text{base}} + \mathcal{A}_{\text{lateral}} \\ &= 2 \cdot 8 + 70 \\ &= 16 + 70 \\ &= 86 \text{ cm}^2\end{aligned}$$

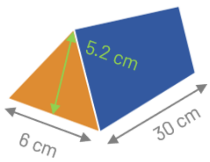
Exercise ML4-2

Calculate the total area of the following right prisms:

Right prism			
Shape of base	Right triangle	Rectangle	Rectangular trapezium
Height			
Perimeter of base			
Base area			
Lateral area			
Total area			

Exercise ML4-3

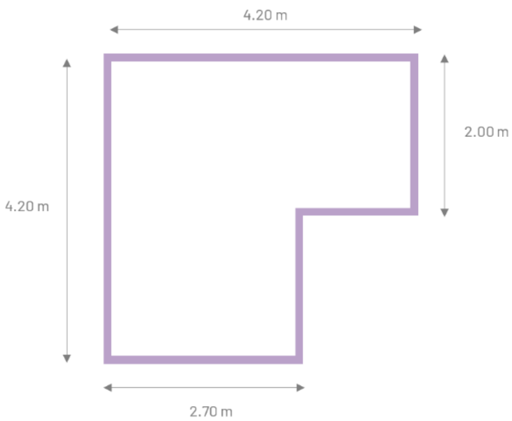
1) A famous type of Swiss chocolate is packaged in a box in the shape of a right prism whose bases are equilateral triangles.



   a) Calculate the total area of the cardboard needed to make this packaging.

   b) How many packages can be made using 1 m^2 of cardboard?

2) Pablo wants to paint the walls and ceiling of his workshop navy blue.
   The walls are 2.5 m high. You can forget about doors and windows when calculating the surface. All the corners of the room are right angles.



   a) Help Pablo calculate the total surface area to be painted.

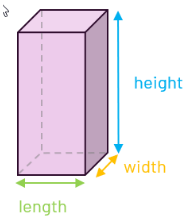
   b) A 2.5 litre pot of paint costs €30 and covers an area of 25 m^2. Help Pablo work out the total cost of the paint.

Mini lesson 5: Volume of a right prism

Use the following application Mathigon to calculate the **volume of** the three **cuboids**.

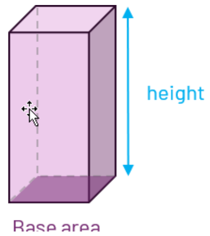
A formula for calculating the **volume** of a **rectangular cuboid** is:

$$\mathcal{V}_{rectangular\ cuboid} = \text{length} \cdot \text{width} \cdot \text{height}$$

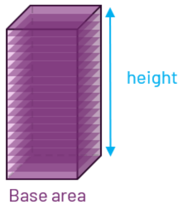


You can also rewrite this formula using **the area of** the rectangular **base**:

$$\begin{aligned}\mathcal{V}_{\text{pavé droit}} &= \underbrace{\text{length} \cdot \text{width}}_{=\mathcal{A}_{\text{base}}} \cdot \text{height} \\ &= \mathcal{A}_{\text{base}} \cdot \text{height}\end{aligned}$$



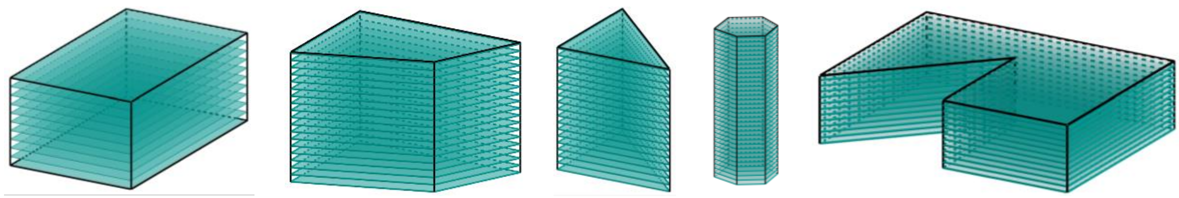
This formula can be explained using the following mental image: imagine the area of the base as an extremely thin sheet of paper of zero thickness. Then, in order to fill the entire right-angled block, you place several of these sheets on top of each other, each with an area identical to that of the base. How many will it take to fill the entire volume of the block? As many as the height of the solid. So, multiply the area of the base by the height and you get the volume of the right block.



In fact this formula can also be used to calculate the volume of any right prism. In general, you can remember that :

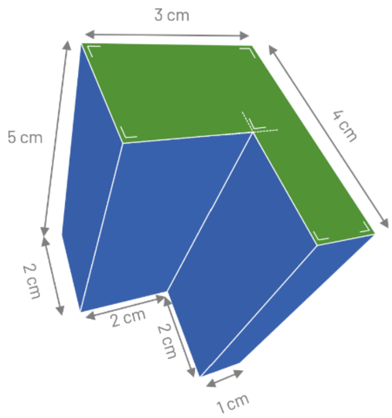
The **volume** of a right prism is equal to the **product of the area of the base** and the **height of** the prism.

$$\mathcal{V}_{\text{right prism}} = \mathcal{A}_{\text{base}} \cdot \text{height}$$



Example:

Consider the following right prism.



- The area of the base is :

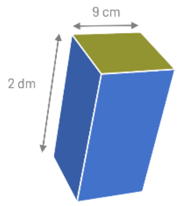
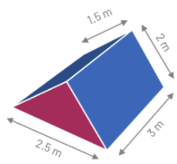
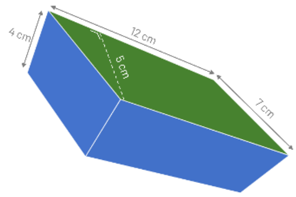
$$\begin{aligned}\mathcal{A}_{\text{base}} &= 4 \cdot 3 - 2 \cdot 2 \\ &= 12 - 4 \\ &= 8 \text{ cm}^2\end{aligned}$$

- The volume is :

$$\begin{aligned}\mathcal{V} &= \mathcal{A}_{\text{base}} \cdot \text{height} \\ &= 8 \cdot 5 \\ &= 40 \text{ cm}^3\end{aligned}$$

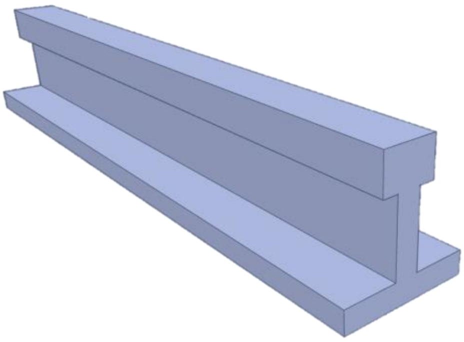
Exercise ML5-1

Calculate the volume of the following right prisms:

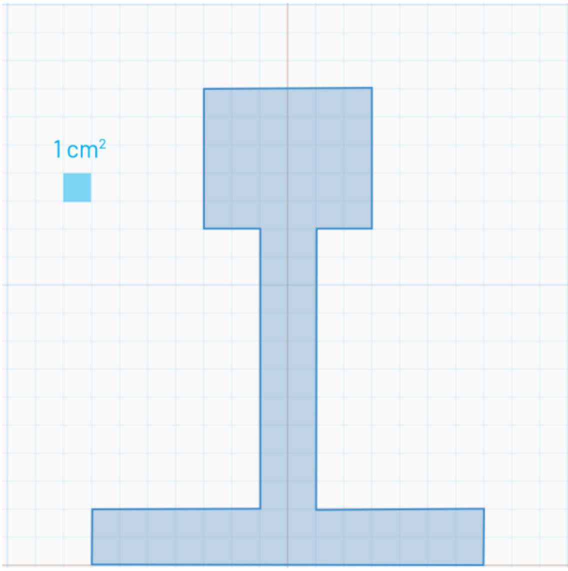
Right prism			
Shape of the base	Square	Right-angled triangle	Parallelogram
Height			
Area of base			
Volume			

Exercise ML5-2

According to the Arcelor-Mittal product catalogue[8] , one metre of type UIC60 rail weighs 60.21 kg. Check this value using the drawings below, knowing that the density of the steel used for railway rails is 7500 kg/m^3.



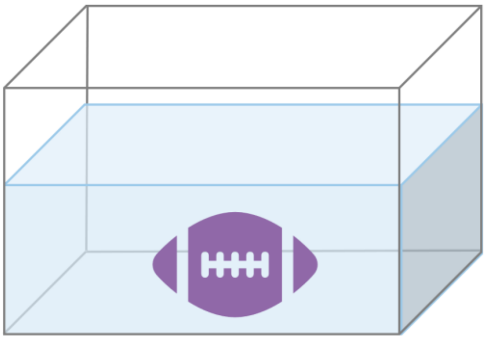
Railway rail (simplified model)



Cross-section

Exercise ML5-3

To determine the volume of an American football, Jerry fills his old fish tank with water to a height of 20 cm. The base of the aquarium is a rectangle measuring 60 cm by 20 cm. He then submerges the football completely in the water. He notices that the water level rises to 23.8 cm from the bottom of the aquarium.



1) Help Jerry calculate the volume of the football.

2) Unfortunately, when Jerry takes the football out of the water, his smartphone falls into the fish tank. He has the latest model which measures 160 mm x 80 mm x 10 mm. Help Jerry to calculate the new water level in the aquarium.

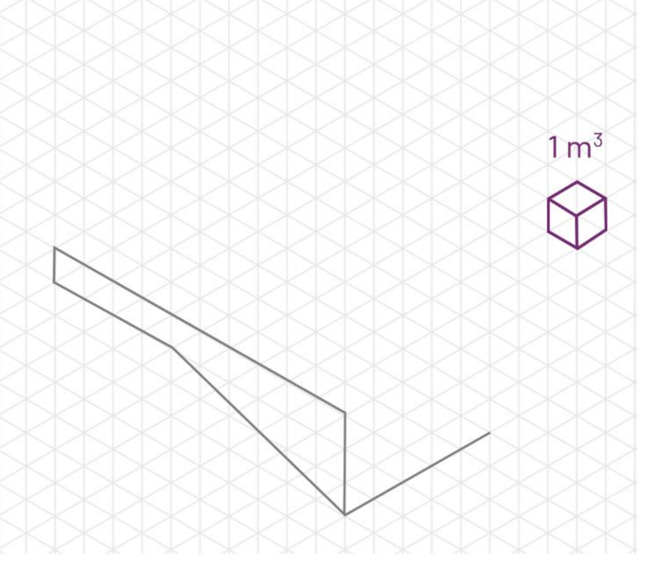
[8] https://rails.arcelormittal.com/wp-content/uploads/2023/10/ArcelorMittal-Transport-Rails-EN.pdf

Exercise ML5-4

Mr PITT has asked an architect to plan an outdoor swimming pool for his new house.

Mr PITT has specific visions for the shape of his new swimming pool. He wants his pool to meet the following requirements:

- The water surface is rectangular, measuring 10 m by 6 m. The vertical walls are perpendicular to the bottom of the pool.
- At the start of the pool, there will be a constant depth of 1 m over the entire width and a length of 4 m.
- After 4 m, the depth increases steadily to reach a maximum depth of 3 m at the other end of the pool.

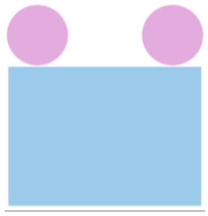
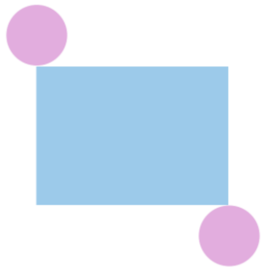


1) The architect has already started to draw a 3D plan of the pool for Mr PITT's swimming pool. Help him to finalise the drawing and enter all the relevant measurements.
2) According to local regulations, it is forbidden to build private swimming pools with a capacity of more than 1000 hl (1 hl = 100 l) of water. Check whether Mr PITT's pool will be granted planning permission.
3) To ensure that the pool is watertight, a plastic film will be placed on the bottom and walls of the pool. Gives a reasonable estimate of the surface area of waterproof film required.
4) Once completed, the pool will be filled using a regular tap with a flow rate of 12 L/min. Calculate the time needed to fill the pool.

Mini lesson 6: Right cylinders

A can of *Pink Horse* is shaped like a **right cylinder**.

Among the proposed nets, mark by ✅ those that can correspond to the net of the cylinder. Mark by ❌ the nets that cannot be folded to form a cylinder.

Net A	Net B	Net C
		
Net D	**Net E**	**Net F**
		

Complete the text below:

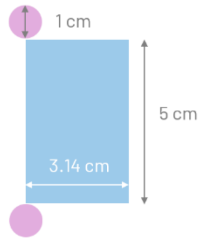
- The nets __________________ correspond to the net of a cylinder.
- The bases of a cylinder are _______________________ and the side face is _______________________ .

In general, you can remember that :

The **side of** a right cylinder is a **rectangle**. The two **bases of** a cylinder **are discs of the same radius**.

Exercise ML6-1

Here are the nets of three right cylinders. Match the nets to the cylinders and fill in the missing measurements:

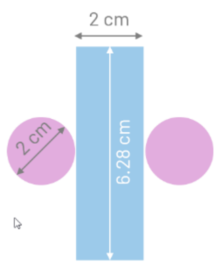


Base radius :

$r =$ ___________

Height :

$h =$ ___________

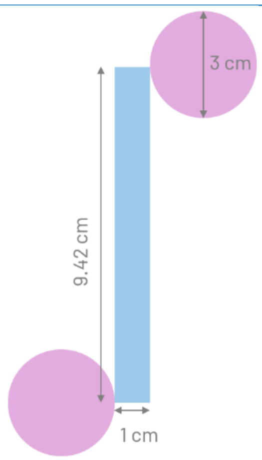


Base radius :

$r =$ ___________

Height :

$h =$ ___________



Base radius :

$r =$ ___________

Height :

$h =$ ___________

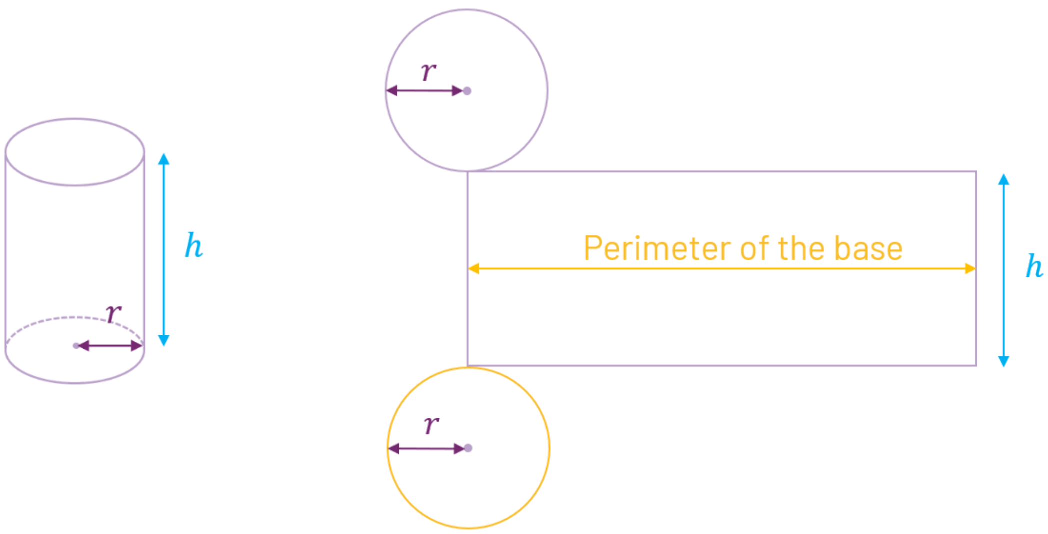
What does the 3rd measurement shown, which does not represent the diameter of the base or the height of the cylinder, represent?

- The radius of the base is equal to _________________ of the diameter of the base.
- The height of the rectangular side is ________________ of the cylinder.
- The length of the rectangular side is _________________ of the base.

In general, you can assume that :

For a right cylinder with radius r and height h :

- The **side face** is a rectangle of height $\boldsymbol{h}$ and length equal to the **perimeter of the base**.
- The two **bases** are discs of radius $\boldsymbol{r}$.



Mini lesson 7: Area and volume of a cylinder

The principles we have seen for calculating the area and volume of a right prism also apply to a cylinder.

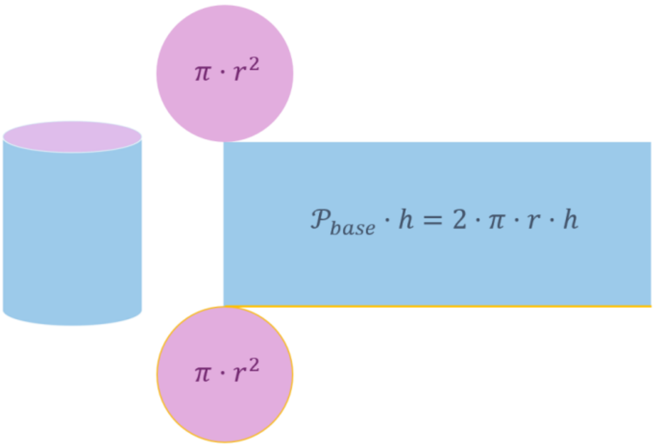
In general, you can remember that :

- The **area** of the **side of** a cylinder is equal to the **product of** the **perimeter** of the **base** and the **height of** the cylinder.

$$\mathcal{A}_{\text{lateral}} = \mathcal{P}_{\text{base}} \cdot \text{height}$$

- The **total area** of a cylinder is equal to the **sum** of the **area** of the two **bases** and ***the* lateral area**.

$$\mathcal{A}_{\text{total}} = 2 \cdot \mathcal{A}_{\text{base}} + \mathcal{A}_{\text{lateral}}$$



Lateral area of a right cylinder :

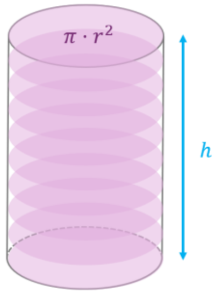
$$\begin{aligned}\mathcal{A}_{\text{lateral}} &= \mathcal{P}_{\text{base}} \cdot \text{height} \\ &= 2 \cdot \pi \cdot r \cdot h \\ &= 2\pi r h\end{aligned}$$

Total area of a right cylinder :

$$\begin{aligned}\mathcal{A}_{\text{total}} &= 2 \cdot \mathcal{A}_{\text{base}} + \mathcal{A}_{\text{lateral}} \\ &= 2 \cdot \pi \cdot r^2 + 2 \cdot \pi \cdot r \cdot h \\ &= 2\pi r^2 + 2\pi r h\end{aligned}$$

- The **volume** of a **cylinder** is equal to the **product of the area of the base** and the **height of** the cylinder.

$$\mathcal{V}_{\text{cylinder}} = \mathcal{A}_{\text{base}} \cdot \text{height}$$

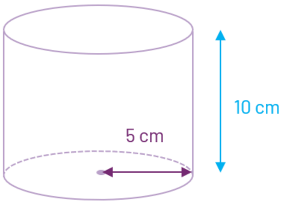
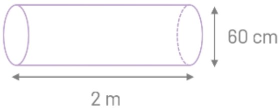
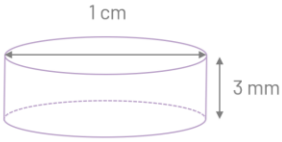


Volume of a right cylinder:

$$\begin{aligned}\mathcal{V}_{\text{cylinder}} &= \mathcal{A}_{\text{base}} \cdot \text{height}\\ &= \pi \cdot r^2 \cdot h\\ &= \pi r^2 h\end{aligned}$$

Exercise ML7-1

Calculate the volume and total area of the following right cylinders:

Cylinder			
Area of base			
Perimeter of base			
Lateral area			
Total area			
Volume			

Exercise ML7-2

We want to compare the volume and total area of three objects:

Object 1	Object 2	Object 3
A 50 cent coin	One CR2032 battery	A single strand of spaghetti
		
Ø 24 mm ; h 2.4 mm	Ø 20 mm ; h 3.2 mm	Ø 2 mm ; h 25 cm

Without doing any **calculations**, try to rank the volume and total area of these objects in ascending order:

$$\mathcal{V}_{.....} \quad < \quad \mathcal{V}_{.....} \quad < \quad \mathcal{V}_{.....}$$

$$\mathcal{A}_{.....} \quad < \quad \mathcal{A}_{.....} \quad < \quad \mathcal{A}_{.....}$$

Check your answer by assuming that these objects are right cylinders and **doing some calculations**. Adjust your classification if necessary:

$$\mathcal{V}_{.....} \quad < \quad \mathcal{V}_{.....} \quad < \quad \mathcal{V}_{.....}$$

$$\mathcal{A}_{.....} \quad < \quad \mathcal{A}_{.....} \quad < \quad \mathcal{A}_{.....}$$

Exercise ML7-3

1) Calculate the volume of PVC needed to produce a DN160 pipe with a length of 2 m, an external diameter of 160 mm and a wall thickness of 4 mm.

2) A wooden plate measuring 100 cm x 100 cm x 18 mm has 190 holes with a diameter of 8 mm. Calculate the weight of the perforated plate, if you know that the wood used has a density of 700 kg/m^3.

3) Calculate the diameter of a can of peeled tomatoes, knowing that the can will contain 500 ml and that the height of the can is 11 cm.

Mini lesson 8: Dice and probabilities

Go to mathigon.com and simulate 500 throws of a D4 die. Count the number of times the die shows a 1, 2, 3 or 4. Then divide the number of times each result appears by 500 to calculate the frequency.

500 throws	1	2	3	4	Total
					
Frequency					

You can see that the frequencies are all approximately equal to _________.

For a D4 die, you can conclude that the theoretical frequency of getting a 1, 2, 3 or 4 is __________________ .

Repeat the experiment with a D6 die using the following link:

500 throws	1	2	3	4	5	6	Total
							
Frequency							

You can see that the frequencies are all approximately equal to _________.

For a D6 die, you can conclude that the theoretical frequency of getting a 1, 2, 3, 4, 5 or 6 is __________________ .

Complete the following sentences:

The theoretical frequency of getting a 7 with a D8 is __________________ .

The theoretical frequency of obtaining a 10 with a D10 is __________________ .

The theoretical frequency of obtaining a 3 with a D20 is __________________ .